

Homework: Feb 22, 2022. Number Theory.

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3-4      5

Prove that  $x/(1 - x - x^2)$  is the generating function for the sequence of Fibonacci numbers.

4-1      2 a-b-c      5

# 2. Do there exist integers  $x$  such that (a)  $6x \equiv 5 \pmod{4}$ , (b)  $10x \equiv 8 \pmod{6}$ , (c)  $12x \equiv 9 \pmod{6}$ ?

# 5. Prove that if  $|a| < k/2$ ,  $|b| < k/2$ , and  $a \equiv b \pmod{k}$ , then  $a = b$ .

4-2      3

Suppose  $\{a_1, a_2, \dots, a_k\}$  is a complete residue system modulo  $k$ , where  $k$  is a prime. Prove that for each integer  $n$  and each nonnegative  $s$  there exists a congruence of the form

$$n \equiv \sum_{j=0}^s b_j k^j \pmod{k^{s+1}}$$

where each  $b_j$  is one of the  $a_i$ .