201 Linear Algebra, Practice Midterm2

Duration: 50 mins

1.
$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

Find the matrix of the transformation $T(\vec{x}) = A\vec{x}$ with respect to the basis $\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \}$.

2. $T(M) = M \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ defines a linear transformation on the space of 2×2 matrices; $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$.

   (a) Write down the matrix for this transformation in terms of the standard basis for $\mathbb{R}^{2\times 2}$

   (b) Find the Kernel and Image of T.

   (c) Find the matrix of T with respect to the basis $\mathcal{B} = \left(\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \right)$.

   (d) How is the matrix in part (c) related to the matrix in part (a)?

3. Find an orthonormal basis for the subspace of $\mathbb{R}^4$ consisting of all those vectors that are perpendicular to $\begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$.

4. Find the least-squares solution $\vec{x}_*$ of the system $\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$.

5. TRUE or FALSE. Justify your answer.

   (a) All linear transformations from P_3 to $\mathbb{R}^{2\times 2}$ are isomorphisms

   (b) If the matrix of a linear transformation $T : V \to V$ with respect to some basis is invertible, then T is invertible.

   (c) If the 2×2 matrix R represents the reflection about a line in $\mathbb{R}^2$, then there is an invertible 2×2 matrix S such that $R = S^{-1} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} S$.

   (d) If a matrix A is similar to B, and A is orthogonal, then so is B.

   (e) If the matrix A is orthogonal then A^3 is orthogonal.