LINEAR ALGEBRA
First Midterm Exam

JOHNS HOPKINS UNIVERSITY
SPRING 2013

You have **50 MINUTES**.
No calculators, books or notes allowed.

Academic Honesty Certificate. I agree to complete this exam without unauthorized assistance from any person, materials or device.

Signature: ________________________ Date: ____________

Name: ________________________ Section №: ____________
(or TA's name)

Question	*Score*
1	
2	
3	
4	
5 (bonus)	

(1) (a) [15 points] Find all solutions to the system of equations:

$$\begin{cases} x & + & y & + & 6z & = & 8 \\ 2x & + & 3y & + & 16z & = & 21 \end{cases}$$

using Gaussian elimination. Is the system consistent? Why?

(b) [10 points] Does the system of equations:

$$\begin{cases} 2x & + & & & 12z & = & 14 \\ & & 2y & + & 16z & = & 18 \\ x & + & 2y & + & 22z & = & 25 \end{cases}$$

have a unique solution? Justify your answer.

(2) [25 points] Let:

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 5 \\ 1 & 1 & 1 & 15 \end{bmatrix}$$

Can an equation:

$$A\vec{x} = \vec{b}$$

have infinitely many solutions while another equation:

$$A\vec{x} = \vec{c}$$

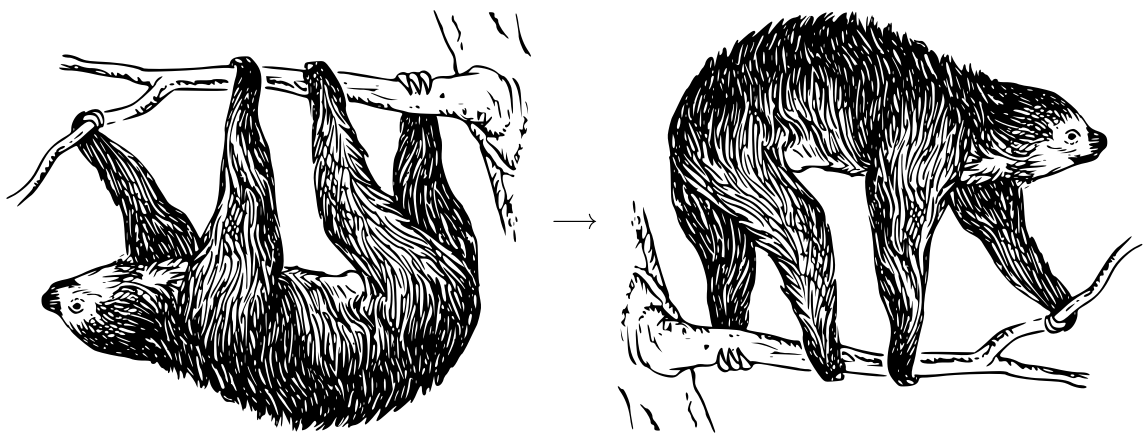
has none whatsoever? If no, explain why not. If yes, find vectors $\vec{b}$ and $\vec{c}$ in $\mathbf{R}^4$ for which this is true.

(3) (a) [5 points] Compute the matrix products BA and AB where:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(b) [10 points] Does the matrix A above have an inverse? If yes, compute it. If no, why not?

(3) (c) [10 points] Write down the matrix for the following linear transformation. (The origin is at the center of each drawing.) Explain how you reached your answer.



[This must be how sloths look to one another!]

(4) (a) [5 points] What does it mean to say that vectors $\vec{v}_1, \ldots, \vec{v}_n$ are *linearly independent?*

(b) [5 points] Are these vectors linearly independent? Justify your answer using determinants.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

(4) (c) [15 points] What is the dimension of the space spanned by the following vectors? Explain your approach and show your work.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} \qquad \vec{v}_4 = \begin{bmatrix} 1 \\ -1 \\ 3 \\ 2 \\ 3 \end{bmatrix}$$

(5) [20 bonus points] Find a basis for the image of the linear transformation:

$$A = \begin{bmatrix} a & a & b & a \\ a & a & b & 0 \\ a & b & a & b \\ a & b & a & 0 \end{bmatrix}$$

for any real numbers a and b.

[*Hint: The special cases* $(a,b) = (0,0)$ *and* $(a,b) = (0,1)$ *immediately show that the number of basis vectors will depend on the values* a *and* b *take, so carry out as much row reduction as possible without dividing by possibly vanishing numbers and break into cases at the last step.*]