LINEAR ALGEBRA (MATH 110.201)

MIDTERM II - 1 APRIL 2016

Name: ______________________________

Section number/TA: ______________________________

Instructions:

(1) Do not open this packet until instructed to do so.
(2) This midterm should be completed in **50 minutes**.
(3) Notes, the textbook, and digital devices **are not permitted**.
(4) Discussion or collaboration is **not permitted**.
(5) All solutions must be written on the pages of this booklet.
(6) Justify your answers, and write clearly; **points will be subtracted otherwise**.
(7) Once you submit your exam, you will not be allowed to modify it.
(8) By submitting this exam, you are agreeing to the above terms. Cheating or refusing to stop writing when time is called may result in an automatic failure.

Exercise	Points	Your score
1	8	
2	12	
3	16	
4	12	
5	12	
6	12	
7	12	
8	16	
Total	100	

Exercise 1 (8 points)**.** *Give an example of a* 2×2 *matrix* A *with* $\mathrm{Im}(A) \neq \mathrm{Im}(\mathrm{RREF}(A))$.

Solution:

Exercise 2 (12 points)**.** *Suppose that B is any $m \times n$ matrix having only $\vec{0}_n$ in its kernel, and suppose that C is any $n \times k$ matrix having only $\vec{0}_k$ in its kernel. Under these assumptions, can you describe all vectors in* $\operatorname{Ker}(BC)$*?*

Solution:

Exercise 3 (16 points)**.** *Consider the following matrix:*

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 2 & 3 \\ 2 & 3 & 4 & 2 & 3 & 4 \\ 3 & 4 & 5 & 3 & 4 & 5 \\ 4 & 5 & 6 & 4 & 5 & 6 \end{pmatrix}$$

(1) *(4 points) Compute* $\mathrm{RREF}(A)$.

(2) *(6 points) Give a basis of* $\mathrm{Ker}(A)$*, and write down* $\dim(\mathrm{Ker}(A))$.

(3) *(6 points) Give a basis of* $\mathrm{Im}(A)$*, and write down* $\dim(\mathrm{Im}(A))$.

Solution:

Exercise 4 (12 points)**.** *If A is a nonzero 2×5 matrix, can we have* $\dim(\mathrm{Ker}(A)) = 1$*? What are the possibilities for* $\dim(\mathrm{Ker}(A))$*?*

Solution:

Exercise 5 (12 points)**.** *Determine whether the following vectors are a basis of* $\mathbb{R}^4$ *(justify your answer):*

$$\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \quad \begin{bmatrix}1\\2\\2\\2\end{bmatrix}, \quad \begin{bmatrix}1\\2\\3\\3\end{bmatrix}, \quad \begin{bmatrix}1\\2\\3\\4\end{bmatrix}$$

Solution:

Exercise 6 (12 points)**.** *Let* $\mathscr{B}$ *denote the basis* $\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]$ *of* $\mathbb{R}^2$*. Suppose that* $T : \mathbb{R}^2 \to \mathbb{R}^2$ *is a linear transformation whose matrix with respect to* $\mathscr{B}$ *is* $\left[\begin{smallmatrix}a & b\\c & d\end{smallmatrix}\right]$*. What is the matrix of* T *with respect to the standard basis of* $\mathbb{R}^2$*? Express your answer in terms of* a, b, c, d*.*

Solution:

Exercise 7 (12 points)**.** *Suppose that* v_1, v_2, v_3, v_4 *are linearly independent vectors in a linear space* V*. Show that the vectors*

$$v_1, \quad v_2 + v_1, \quad v_3 + v_2 + v_1, \quad v_4 + v_3 + v_2 + v_1$$

are also linearly independent in V*.*

Solution:

Exercise 8 (16 points). *Consider the set $W \subseteq P_3(\mathbb{R})$ of polynomials $f(X)$ of degree ≤ 3 having the property that $f(-2) = 0$.*

(1) *(4 points) Give an example of a degree 3 polynomial in W. Give an example of a degree 3 polynomial which is not in W.*
(2) *(6 points) Show that W is a subspace of $P_3(\mathbb{R})$.*
(3) *(6 points) Find a basis of W.*

Solution: