

## Concept and theory

Every freshman taking a math course must face the fact that they are not going to be taught how to do “the problems”. What I mean is that the mode of instruction will be different from the high school experience of most students. There, they were shown how to do certain problem types, and then drilled and tested on those. Here, they must learn the concepts behind the methods so that they can see how to approach problems in any new situation, how to judge that the methods of the course apply, and to carry out the solutions. The preceding applies to all math courses, not just the honors courses.

Too many students mislabel the concepts as “[irrelevant] theory”. What theory really means is the mathematical justification for the role of the concepts, and that involves abstract theorems and their proofs. While it is sometimes possible to manage without knowing the proofs (see the first paragraph), the structure of the proof can guide you in understanding conceptual distinctions.

There is a good example of the distinction between concept and theory, coming from Calculus II (as done in a college). I’m referring to the notion of the convergence of infinite series. (Go to a textbook if necessary to refresh your memory.)

*Concepts:* (1) An infinite series can either converge or diverge. (2) There is the Ratio Test for convergence of a series with positive terms (underlying it is a comparison with a geometric series). It involves the limit of a ratio. (3) The Ratio Test is inconclusive when the ratio limit is equal to one; know examples of cases where the series in question converges, diverges. (4) There is also the Root Test for series with non-negative terms.

*Theory:* [(1) Understand the definition of convergence.] (2) Give a proof that explains why the Ratio Test and Root Test work (BTW, a good Calculus textbook will contain the proofs of both). (3) It is a fact that whenever the Ratio Test gives an outcome, the Root Test gives the same—**of course!**—outcome; the Root Test works sometimes when the Ratio Test fails. (Know an example.) Therefore, one can give a proof of this fact; it is beyond the scope of most Calculus books. (Thus, the Root Test is theoretically better than the Ratio Test, though sometimes it is more convenient to use the more limited Ratio Test.)