201 Linear Algebra, Practice Midterm

Duration: 50 mins

1. Solve the system

$$x + 2y + 3z = 1$$
$$3x + 4y + 7z = 1$$
$$5x + 6y + 11z = 1$$

   Determine the rank of the coefficient matrix.

2. Determine if the columns of the coefficient matrix in question 1. are linearly independent. If not, find all possible linear dependency relations among them.

3. Show that the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ y \\ x \end{pmatrix}$$

   is an invertible linear transformation. Find the matrix associated to T^{-1}.

4. Let $L = \{t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \mid t \in \mathbb{R}\}$ be a line in $\mathbb{R}^3$. Determine the matrix of orthogonal projection onto this line. Describe the image and kernel of this matrix.

5. True or False. Justify your answer.

   (a) There is a 4×4 matrix of A of rank 3 such that the system $A\vec{x} = \vec{0}$ has a unique solution.

   (b) A system with 4 equations and 3 unknowns is always inconsistent.

   (c) If the matrices A and B are both invertible, then $A + B$ must be invertible as well.

   (d) There exists a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(\vec{e}_1) = 5\vec{e}_2$ and $T(5\vec{e}_1) = \vec{e}_2$.