

October 2, 2009

Name .....  
Section/ Name of your TA .....

MIDTERM EXAM 1 *35pts.*  
MATH 201 VER \*\*\*

- There are 10 pages in the exam including this page.
- Write all your answers clearly. You have to show work to get points for your answers.
- Read all the questions carefully.
- You can write on both sides of the paper. Indicate that the answer follows on the back of the page.
- Use of Calculators is *not* allowed during the exam.

(1) ..... /3

(2) ..... /4

(3) ..... /5

(4) ..... /4

(5) ..... /3

(6) ..... /5

(7) ..... /4

(8) ..... /3

(9) ..... /4

Total ..... /35

2

- (1) *3pts.* Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a function . What does it mean for  $T$  to be a linear transformation?

(2) *4pts.* Find the orthogonal projection of the vectors  $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  and  $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$  onto the line spanned (determined) by the vector  $\vec{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ .

4

- (3) *5pts.* Find the matrix  $A$  representing the linear transformation  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  that reflects a vector along the line  $x - 2y = 0$ . Show work.

- (4) *4pts* Write down  $A^{-1}$  **without performing any computations**. Give reasons for your answer. The matrix  $A$  here is the matrix you found in the Problem 3.

(5) *3pts.* Find the volume of the parallelopiped determined by the vectors

$$\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \vec{v} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \text{ and } \vec{w} = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}.$$

(6) 5pts. Let  $B = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 1 \\ 4 & -2 & -3 \end{bmatrix}$ . Find all  $\vec{x} \in \mathbb{R}^3$  such that  $B\vec{x} = \vec{0}$ .

(7) *4pts.* Define the Rank of a  $m \times n$  matrix.

For what values of  $k$  does the system  $B\vec{x} = \begin{bmatrix} 1 \\ 0 \\ k \end{bmatrix}$  have a solution?

The matrix  $B$  here is same as that in Problem 6.



- (8) *3pts.* Let  $\vec{v} = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}$ . Given that  $\vec{v}$  is a solution to the following system, find all possible solutions of the following system of equations **without any computations**. Explain your answer.

$$\begin{aligned} 2x & \quad - z = 1 \\ -x + y + z & = 0 \\ 4x - 2y - 3z & = 1 \end{aligned}$$

(9) *4pts.* Find the Adjoint of the matrix  $B$  if it exists.

Is  $B$  invertible? Explain your answer. The matrix  $B$  here is same as that in Problem 6.