SAMPLE MIDTERM 2 100pts.

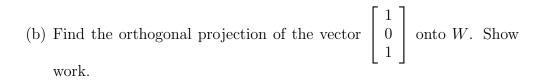
- There are 6 pages in the exam including this page.
- Write all your answers clearly. You have to show work to get points for your answers.
- You can write on both sides of the paper. Indicate that the answer follows on the back of the page.
- Use of Calculators is *not* allowed during the exam.
- $(1) \ldots /22$
- $(2) \ldots /22$
- $(3) \ldots /20$
- $(4) \ldots /36$

Total/100

- (1) 22pts. Let \mathcal{P}_2 denote the set of all polynomials of degree less than or equal to 2. Let $T : \mathcal{P}_2 \to \mathcal{P}_2$ be a linear transformation defined as T(f(t)) = f(t-1), where f(t) is a polynomial of degree less than or equal to 2.
 - (a) Find the matrix of this transformation with respect to the basis $\{1, t, t^2\}$ of \mathcal{P}_2 . Show work.

(b) Evaluate the determinant of the matrix you found in part (a). Show work.

(2)22 pts. Let $\left\{ \begin{bmatrix} 1\\1\\0 \end{bmatrix}, \begin{bmatrix} 0\\1\\1 \end{bmatrix} \right\}$ be a basis of the subspace W of \mathbb{R}^3 . (a) Find a orthonormal basis of W. Show work.



- (3) 20pts. Let \mathbb{C} denote the set of complex numbers $\{a + bi : a, b \in \mathbb{R}\}$. Then both $\mathcal{B}_1 = \{1, i\}$ and $\mathcal{B}_2 = \{1 + i, 1 - i\}$ are bases for \mathbb{C} . (a) What is the matrix that transforms a vector in \mathcal{B}_1 coordinates into a
 - matrix in \mathcal{B}_2 -coordinates? Show work.

(b) Write down the element 4 + 2i in \mathcal{B}_2 -coordinates. Show work.

(4) 16pts. Give short answers to the following.

(a) Let det
$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -3$$
. Compute det $\begin{bmatrix} a+d & b+e & c+f \\ 2g & 2h & 2i \\ d & e & f \end{bmatrix}$.
Give reasons to support your answer.

(b) Let A be a 2×2 matrix such that detA = -1 then A is orthogonal. State true or false with justification. (c) Let u, v, w be vectors in \mathbb{R}^n . Let w be orthogonal to both u and v. Then u + v is orthogonal to 3w. State true or false with justification.

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