Name ....

PRACTICE EXAM 1 *40pts.*

- **There are 6 pages in the exam including this page.**
- **Write all your answers clearly. You have to show work to get points for your answers.**
- **You can write on both sides of the paper. Indicate that the answer follows on the back of the page.**
- **Use of Calculators is *not* allowed during the exam.**

(1) ..../10

(2) ..../10

(3) ..../8

(4) ..../12

Total ..../40

1(a) *7pts.* Find all solutions to the given system of equations.

$$\begin{array}{rcrcrcrcl} x_1 & & & + & x_3 & + & x_4 & = & 0 \\ 2x_1 & + & x_2 & & & - & x_4 & = & 0 \\ x_1 & - & x_2 & + & 2x_3 & + & 2x_4 & = & 0 \\ x_1 & + & x_2 & + & 3x_3 & + & 6x_4 & = & 0 \end{array}$$

1(b) *3pts.* Are the vectors $\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 6 \end{bmatrix}$ linearly independent? Why or why not?

2(a) *7pts.* Let $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. Compute the inverse of A, if it exists.

2(b) *3pts.* Find all possible solutions for the system

$$\begin{array}{rcrcrcr} 2x_1 & + & x_2 & - & x_3 & = & 2 \\ x_1 & + & x_2 & + & x_3 & = & 3 \\ 2x_1 & + & 2x_2 & + & 3x_3 & = & -1 \end{array}$$

(3) *8pts.* Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 2 \\ 1 \\ h \end{bmatrix}$.

(a) *6pts.* Find all values of h such that $v = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ in the Span$\{v_1, v_2, v_3\}$?

(b) *2pts.* Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined as $T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = x_1v_1 + x_2v_2 + x_3v_3$. For what values of h is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is in ImT.

(4) *12pts.* State True or False with justification. *3pts. each for the justification.*

(a) The linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(v) = v + v$ is a linear transformation.

(b) If A, B and C are 2×2 invertible matrices then, $(AB)C$ is invertible.

4(c) The vectors $\begin{bmatrix} 4 \\ -8 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ -12 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 4 \\ -1 \end{bmatrix}$, span $\mathbb{R}^3$.

4(d) The matrix $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ represents reflection about a line.

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