110.201 Linear Algebra
6th Quiz

May 5, 2005

Problem 1 Let

$$A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}.$$

1. Find an invertible matrix S and a diagonal matrix D such that $S^{-1}AS = D$.
2. Find a formula for the entries of A^n, where n is a positive integer.

Problem 2 Let

$$A = \begin{bmatrix} 0 & -i \\ 2i & 1 \end{bmatrix}, \qquad i^2 = -1.$$

1. Find the eigenvalues of A.

2 Find A^{30}.

Problem 3 Let A to be a 3×3 matrix with eigenvalues $1, -1, 2$.
Let $B = A^3 - 5A^2$.

1. Find the eigenvalues of B. Is B diagonalizable? If yes, find a diagonal matrix D such that $D = S^{-1}BS$, for some orthogonal matrix S.
2. Find $\det(B)$ and $\det(A - 5I_3)$.