110.201 Linear Algebra
5th Quiz

April 21, 2005

Problem 1 Find the determinant of the $n \times n$ matrix

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Problem 2 Let A be an $n \times n$ matrix obeying the equation

$$A^2 = A.$$

a) What are the possible values of $\det(A)$? Why?

b) Let V be the image of A and $m = \dim V$ Find all relationships between m, n and the values for $\det(A)$ you found above
[An acceptable statement would be something like "If $\det(A) = \ldots$, then ..."].
Can $m = n$? If so, and $m = n$, what can you say about A?

Problem 3 Suppose that two square matrices satisfy the following identity $AB = -BA$. Find the flaw in the following argument, showing a counterexample:

Taking determinants gives $(\det A)(\det B) = -(\det B)(\det A)$, so either A or B must have zero determinant. Thus $AB = -BA$ is only possible if A or B is singular.