110.201 Linear Algebra
5th Quiz

April 22, 2005

Problem 1 Using determinant rules, find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 4 & 4 & 4 \\ 1 & 4 & 9 & 9 \\ 1 & 4 & 9 & 16 \end{bmatrix}.$$

Problem 2 True or false, with reason if true and counterexample if false:

1. If A and B are identical except in the upper-left corner, where $b_{11} = 2a_{11}$, then $\det B = 2 \det A$.
2. The determinant of a matrix is the product of the pivots.
3. If A is invertible and B is singular, then $A + B$ is invertible.
4. If A is invertible and B is singular, then AB is singular.

Problem 3 An invertible linear map $L : \mathbb{R}^n \to \mathbb{R}^n$ is called orientation preserving if $\det(A) > 0$, and orientation reversing otherwise.

a) Let $T_n(\underline{x})$ be the opposite of the identity map in $\mathbb{R}^n$, i.e.

$$T_n(\underline{x}) = -\underline{x}.$$

Is T_n orientation preserving or orientation reversing?

b) Prove that for any invertible map $L : \mathbb{R}^n \to \mathbb{R}^n$, the map $LL^t : \mathbb{R}^n \to \mathbb{R}^n$ is orientation preserving.

c) The linear map $\mathrm{ref}_V : \mathbb{R}^3 \to \mathbb{R}^3$ takes a vector $\underline{x}$ to its reflection $\mathrm{ref}_V(\underline{x})$ in a two-dimensional subspace $V \subset \mathbb{R}^3$. Is ref_V orientation preserving or reversing? Does the answer depend on V?

[Hint: Try to find a basis in which the matrix for ref_V is simple].