110.201 Linear Algebra
4th Quiz

April 7, 2005

Problem 1 Find an orthogonal basis for the plane

$$x - y + z = 0,$$

viewed as a subspace of $\mathbb{R}^3$.

Problem 2 Let $\vec{e}_1, \vec{e}_2, \vec{e}_3$ be the standard basis of $\mathbb{R}^3$. Consider the plane V spanned by $\vec{e}_1$ and $\vec{e}_2$.

a. For a given vector $\vec{w} = (a, b, c) \in \mathbb{R}^3$, calculate the vector $\vec{u} \in V$ that minimizes the distance between V and $\vec{w}$, i.e. find $\vec{u} \in V$ such that

$$\|\vec{u} - \vec{w}\| \leq \|\vec{v} - \vec{w}\| \quad \forall \vec{v} \in V.$$

b. In the inequality above, is such a $\vec{u}$ unique for every $\vec{w}$? If so, is the assignment $\vec{w} \mapsto \vec{u}$ linear? If it is, find the matrix A of this linear transformation in the standard basis of $\mathbb{R}^3$.

Problem 3 Find the $Q - R$ factorization of the matrix

$$\begin{pmatrix} 4 & 6 & 10 \\ 0 & 8 & 12 \\ 0 & 0 & 14 \end{pmatrix}.$$