110.201 Linear Algebra
4th Quiz

April 8, 2005

Problem 1 Let $\vec{e}_i,\ i = 1, 2, 3, 4$ be the standard basis of $\mathbb{R}^4$. Is there an orthogonal matrix A with

$$A\vec{e}_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix},\ A\vec{e}_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix},\ A\vec{e}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},\ A\vec{e}_4 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

or not? If so, find A, and if not, explain.

Problem 2 Find an orthonormal basis for the subspace $V \subset \mathbb{R}^3$ spanned by the vectors

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Moreover, find a normal basis for the orthogonal complement of V.

Problem 3 Let $\{\vec{\alpha}_1, \vec{\alpha}_2, \ldots, \vec{\alpha}_k\}$ be a set of mutually orthogonal vectors in $\mathbb{R}^n$.

1. Show that for any $\vec{v} \in \mathbb{R}^n$, the vector

$$\vec{v} - (\mathrm{proj}_{\vec{\alpha}_1}(\vec{v}) + \mathrm{proj}_{\vec{\alpha}_2}(\vec{v}) + \cdots + \mathrm{proj}_{\vec{\alpha}_k}(\vec{v}))$$

   is orthogonal to each of the $\vec{\alpha}_1, \vec{\alpha}_2, \ldots, \vec{\alpha}_k$.

2. Verify this claim for $\mathbb{R}^3$ with $\vec{\alpha_1} = \vec{e_1}$, $\vec{\alpha_2} = \vec{e_2}$, and letting $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Draw a picture.