110.201 Linear Algebra
3rd Quiz

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Notation.

- P_n = space of polynomials, with real coefficients, of degree at most n.
- $\mathbb{R}^{m\times n}$ = space of m by n real matrices.

Problem 1 Determine whether the following spaces are isomorphic. In case they are isomorphic, define an isomorphism relating them. Justify your answer.

1. $\mathbb{R}^2$ and $\mathbb{R}^4$
2. P_5 and $\mathbb{R}^5$
3. $\mathbb{R}^{2\times 3}$ and $\mathbb{R}^6$
4. P_5 and $\mathbb{R}^{2\times 3}$
5. $\mathbb{R}^{2\times k}$ and $\mathbb{C}^k$, for $k \in \mathbb{N}$

Problem 2 Let $V = \mathcal{C}^1([0,1])$ be the set of continuously differentiable functions on the closed interval $[0,1]$. V is a real linear space with respect to the operations of pointwise addition of functions and scalar multiplication.

(a) Prove that the functions $f(x) = \cos x$, $g(x) = 2x$, and $h(x) = e^x$ are linearly independent in V.

(b) Given an integer $n > 0$, find $n+1$ linearly independent elements in V. Prove that your answer is correct.

(c) Is V isomorphic to $\mathbb{R}^m$ for some positive integer m? Why or why not?

Problem 3 Let $T : P_2 \to P_2$ be the linear transformation defined by

$$T(p(t)) = p''(t) + 4p'(t).$$

Find the image and the kernel of T as well as its rank and nullity.