110.201 Quiz 2 Solutions

February 28, 2005

Problem 1 The first part's answer is

$$L = \{t\vec{v} : t \in \mathbb{R}\}$$

To find $\text{proj}_L \vec{w}$ use the formula

$$\text{proj}_L \vec{w} = \frac{1}{\|\vec{v}\|^2}(\vec{w} \cdot \vec{v})\vec{v} = \frac{31}{49}\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$$

Problem 2 We first reduce A to rref:

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 3 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow[(III)+(II)^*]{} \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 4 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow[2(III)^*+(I)]{} \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\xrightarrow[-2(III)+(II)^*]{} \begin{bmatrix} 2 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow[(1/3)(II)+(I)^*,(III)^*]{} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\xrightarrow[-(1/3)(II)^*,(1/2)((III)^*,(I)^*]{} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since A can be rowreduced to the 3 by 3 identity matrix, it is invertible. Now perform these row operations on the three-dimensional identity matrix to obtain A^{-1}:

$$A^{-1} = \begin{bmatrix} 1/6 & 1/6 & -1/2 \\ 2/3 & -1/3 & 1 \\ 1/6 & 1/6 & 1/2 \end{bmatrix}$$

Problem 3 V is the plane orthogonal to the unit vector $\vec{v} = 1/\sqrt{6}[2, -1, 1]$. To find a linear transformation $T_A : \mathbb{R}^3 \to \mathbb{R}$ with $\text{Ker}(T_A) = V$, just take the dot product with $\vec{v}$:

$$T_A(\vec{w}) = \vec{v} \cdot \vec{w}$$

The matrix for T_A is

$$[2/\sqrt{6}\ -1/\sqrt{6}\ 1/\sqrt{6}]$$

To find a linear transformation $T_B : \mathbb{R}^2 \to \mathbb{R}^3$ with $\text{Image}(T_B) = V$, find two linearly independent vectors orthogonal to $\vec{v}$; they will span V. Two such vectors are

$$\vec{w}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{w}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}.$$

The image of $\mathbb{R}^2$ under the 3 by 2 matrix

$$[\vec{w}_1\ \vec{w}_2] = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}$$

is then equal to V.
Alternatively one could have used the projections onto V and $\vec{v}$.