Midterm 2 Review Outline

November 11, 2016

Test format: There are 6 problems in total. The first problem contains 10 True/False questions. The True/False question is like:

Question: 11 vectors in R^{10} can be linearly independent.
Answer: False.

The following HW problems are removed from the review: 30,48,67 in 5.3, 11 in 5.4, 10 in 5.5.

Midterm 2 covers Chapter 4, Chapter 5, Chapter 6 and Section 7.1, 7.2.

Chapter 4

- Section 4.1: Definition of a vector space. You are not required to check whether a set is a vector space.
- Section 4.1: Definition of subspace in a vector space. Given a subset in a vector space, determine whether it is a subspace.
- Section 4.1: Linear independence/dependence, linear combinations, span, basis, dimension and coordinates.
- Section 4.2: Linear transformation: given a transformation between two vector spaces, determine whether it is a linear transformation.
- Section 4.2: Image, kernel, rank and nullity of a linear transformation. Given a linear transformation, find bases of the image and the kernel, determine the rank and nullity. Keep in mind that

$$\text{rank} + \text{nullity} = \dim(\text{input space}).$$

- Section 4.2: Isomorphism: an invertible linear transformation is called an isomorphism. Given a transformation between two vector spaces, determine whether it is an isomorphism.
- Section 4.3: The $\mathfrak{B}$-matrix of a linear transformation (Definition 4.3.1). Given a linear transformation and a basis $\mathfrak{B}$, write down the $\mathfrak{B}$-matrix. Use this matrix to find bases of the image and kernel of the linear transformation. Also use this matrix to determine whether a linear transformation is invertible. See Example 4 in Section 4.3.
- Section 4.3: Change of basis is not on the exam.

Chapter 5

- Section 5.1: Definition of orthonormal vectors. n orthonormal vectors in $\mathbb{R}^n$ form a basis of $\mathbb{R}^n$.
- Section 5.1: Orthogonal projection to a subspace (Theorem 5.1.4). Given a subspace, compute the orthogonal projection onto it (Theorem 5.1.5).

- Section 5.1: Orthogonal complement: Definition 5.1.7 and Theorem 5.1.8.
- Section 5.2: Given m linearly independent vectors $v_1, \dots, v_m$, run Gram-Schmidt to obtain orthonormal vectors $u_1, \dots, u_m$; or given an $n \times m$ matrix with linearly independent columns, compute the QR factorization.
- Section 5.3: Orthogonal transformations and orthogonal matrices. Orthogonal transformations preserve the length of vectors, and the dot product between two vectors. Theorem 5.3.3 and Theorem 5.3.4. Determine whether a linear transformation is an orthogonal transformation, or whether a matrix is an orthogonal matrix. Theorem 5.3.7 and Summary 5.3.8. Properties of the transpose (Theorem 5.3.9).
- Section 5.4: Theorem 5.4.1 and the applications. Theorem 5.4.2 and Theorem 5.4.3. Given an inconsistent linear system, find the least-squares solution (Theorem 5.4.5). Applications of least squares include finding certain functions that best fit given data points.
- Section 5.5: Definition of inner product and inner product spaces. Notice that we focus on $C[-\pi, \pi]$ with the inner product $\langle f, g\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)g(t)dt$. In this space, compute the inner product, norm and orthogonal projection. Find the Fourier series of a given function.

Chapter 6

- Definition of determinant. Notice that in the lecture we define determinant through Laplace expansion in Theorem 6.2.10 (see my lecture notes as well). The definition of determinant in Section 6.1 is not requried.
- Section 6.2: The relation of elementary row operations and determinants (Theorem 6.2.3). Use Gauss-Jordan elimination to compute the determinant (Algorithm 6.2.5).
- Section 6.2: Properties of determinant: Theorem 6.2.1, Theorem 6.2.2, Theorem 6.2.6, Theorem 6.2.7 and Theorem 6.2.8.
- Section 6.3: Geometric interpretation of determinants (Theorem 6.3.6). Use determinant to compute the volume of a parallelepiped. Theo determinant as expansion factor (Theorem 6.3.7).
- Section 6.3: Cramer's rule.

Chapter 7

- Section 7.1: Eigenvectors, eigenvalues and eigenbases (Definition 7.1.2). Diagonalization (Theorem 7.1.3).Summary 7.1.5.
- Section 7.2: Given a matrix, compute the characteristic polynomial, eigenvalues and eigenvectors of the matrix.