Midterm1 – 201, Fall 2016

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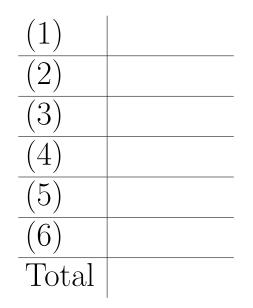
- Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted.
- No notes, books or calculators are allowed.
- Read each problem carefully. Show all work for full credit.
- Make sure you have **10 pages**, including the cover page (Page 1) and the score page (Page 2).

Name: _____

Section:

I would not assist anybody else in the completion of this exam. I would not copy answers from others. I would not have another student take the exam for me.

Signature: _____



Problem 1: (20 points):

Write down T if the following statement is true and F if the statement is false. (2 points each)

Example: <u>F</u> 10 vectors in \mathbb{R}^9 can be linearly independent.

- (1) _____ The transformation $T(A) = A^T$ from $\mathbb{R}^{5 \times 5}$ to $\mathbb{R}^{5 \times 5}$ is an isomorphism.
- (2) _____ Consider the linear transformation:

$$T(\vec{x}) = \det \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \dots & \vec{v}_{n-1} & \vec{x} \end{bmatrix}$$

from \mathbb{R}^n to \mathbb{R} , where $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{n-1}$ are linearly independent vectors in \mathbb{R}^n . The nullity of T is equal to n.

(3) _____ Determinant of the following matrix is necessarily 0.

a_1b_1	a_1b_2	a_1b_3	a_1b_4
a_2b_1	a_2b_2	a_2b_3	a_2b_4
a_3b_1	a_3b_2	a_3b_3	a_3b_4
a_4b_1	a_4b_2	a_4b_3	a_4b_4

(4) _____ The dimension of W^{\perp} is equal to 2, where

$$W = \operatorname{span}\left(\begin{bmatrix} 1\\2\\3\\4 \end{bmatrix}, \begin{bmatrix} -1\\-2\\-3\\-4 \end{bmatrix} \right).$$

(5) _____ The following matrix is an orthogonal matrix.

$$\begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}$$

- (6) _____ If the $n \times n$ matrices A and B are orthogonal matrices, A + B must be orthogonal as well.
- (7) _____ Consider an orthonormal basis $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n$ in \mathbb{R}^n . The least-squares solution of the system $A\vec{x} = \vec{u}_n$ where $A = [\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_{n-1}]$ is $\vec{x} = \vec{0}$.
- (8) (f,g) = f(1)g(1) + f(2)g(2) is an inner product in P_1 .
- (9) Let A be an $n \times n$ matrix. If λ is an eigenvalue of A, then $\lambda^2 + \lambda + 1$ is an eigenvalue of the matrix $A^2 + A + I_n$.
- (10) _____ There is an orthogonal transformation T from \mathbb{R}^3 to \mathbb{R}^3 such that

$$T\begin{bmatrix}2\\3\\0\end{bmatrix} = \begin{bmatrix}3\\0\\2\end{bmatrix}$$
 and $T\begin{bmatrix}-3\\2\\0\end{bmatrix} = \begin{bmatrix}2\\3\\0\end{bmatrix}$.

Problem 2 (12 points):

Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

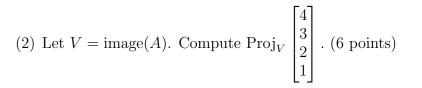
(1) Find the eigenvalues of
$$A$$
. (5 points)

(2) Find the eigenvectors of A. (7 points)

Problem 3 (16 points):

Let

$$A = \begin{bmatrix} -1 & 1\\ 1 & 3\\ 1 & 3\\ -1 & 1 \end{bmatrix}.$$



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Problem 4 (20 points, 5 points each):

Let
$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$
 and $B = \begin{pmatrix} a & d & g \\ b & e & h \\ 1 & 2 & 3 \end{pmatrix}$ such that det $A = 2$
and det $B = 3$.
(1) Compute det $\begin{pmatrix} 0 & 0 & 1 & -2 \\ a & b & c & 1 \\ d & e & f & 2 \\ g & h & i & 3 \end{pmatrix}$.

(2) Suppose
$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
 is the solution of $A\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. Evaluate x_3 .

(3) Let
$$C = (ABA^{-1})^{2016}$$
. Compute det C.

(4) Compute det E, where E is the reduced row echelon form of B.

Problem 5 (20 points):

Consider the linear transformation $T:P_3\to P_3$ such that

T(a + bt + ct² + dt³) = a + dt + (b + c)t² + (a + d)t³.

(1) Given the basis $\mathfrak{B} = \{1, t, t^2, t^3\}$ in P_3 . Find the \mathfrak{B} -matrix of T with respect to the basis \mathfrak{B} . (8 points)

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(2) Find a basis of the kernel of T. (6 points)

(3) Find a basis of the image of T. (6 points)

Problem 6 (12 points):

Let \vec{e} be a unit vector in \mathbb{R}^3 . Consider the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(\vec{x}) = 2(\vec{e} \cdot \vec{x})\vec{e} - \vec{x}$.

(1) Given a geometric interpretation of T. In other words, express T in terms of reflections, projections, or rotations. (4 points)

(2) Is T an isomorphism? Why? (4 points)

(3) Is T an orthogonal transformation? Why? (4 points)

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