Midterm 1 Review Outline

October 1, 2016

Test format: There are 5 problems in total. The first problem contains 10 True/False questions. The True/False question is like:

Question: 11 vectors in R^{10} can be linearly independent.
Answer: False.

Midterm 1 covers Chapter 1, Chapter 2, Chapter 3 and Section 4.1.

Chapter 1

- Section 1.1 and 1.2: Solving a linear system. Given a linear system, write down the augumented matrix, perform elementary row operations on the augumented matrix to solve the linear system.
- Section 1.2 and 1.3: The Reduced Row Echelon Form (RREF) of a matrix; Perform elementary row operations to obtain the RREF of a matrix; Decide whether a linear system has a unique solution, infinitely many solutions, or no solution from the RREF of the augumented matrix.
- Section 1.3: Matrix operations: Matrix addition, scalar multiplication, vector dot product, matrix vector product.
- Section 1.3: Properties of matrix operations.
- Section 1.3: The rank of a matrix.

Chapter 2

- Section 2.1: Definition of linear transformations, Determine whether a transformation is linear or not.
- Section 2.1: Given a linear transformation, write down the matrix representing the linear transformation.
- Section 2.2: Linear transformation in geometry: scaling, orthogonal projection, reflection, rotation.
- Section 2.3: Matrix product: compute matrix multiplication, write matrix product in terms of rows of the first matrix or columns of the second matrix (Theorem 2.3.2). Properties of matrix product. Notice that matrix multiplication is non-commmutative.
- Lecture notes: Definition of one-to-one and onto transformation. Given a linear transformation, determine whether it is one-to-one or onto. A linear transformation $T : R^n \to R^m$ such that $T\mathbf{x} = A\mathbf{x}$ is:

  $$\text{one-to-one} \Leftrightarrow \text{Kernel}(T) = \{\mathbf{0}_{R^n}\} \Leftrightarrow \text{rank}(A) = n.$$

  A linear transformation $T : R^n \to R^m$ such that $T\mathbf{x} = A\mathbf{x}$ is:

$$\text{onto} \Leftrightarrow \text{Image}(T) = R^m \Leftrightarrow \text{rank}(A) = m.$$

- Section 2.4: definition of invertible linear transformations and invertible matrices. Theorem 2.4.3. Find the inverse of a matrix. Properties of inverse.

Chapter 3

- Section 3.1: Definition of kernel. If $T : R^n \to R^m$ is a linear transformation, then Kernel(T) is a subspace of R^n. Given a linear transformation, find its kernel.
- Section 3.1: Definition of image. If $T : R^n \to R^m$ is a linear transformation, then Image(T) is a subspace of R^m. Given a linear transformation, find its image.
- Section 3.2: Definition of subspace. Given a subset of R^n, determine whether it is a subspace.
- Section 3.2: Definition of linear dependence and linear independence. Given a set of vectors, determine whether they are linearly independent.
- Section 3.2: Definition of bases. Given a set of vectors, determine whether they form a basis of R^n. The dimension of a subspace is the number of vectors in the basis.
- Section 3.3: Given a linear transformation, find a basis of the kernel, and the image of the linear transformation, respectively. Find the dimension of the kernel, and the image respectively.
- Section 3.3: If $T : R^n \to R^m$ is a linear transformation, then

$$\dim \text{kernel}(T) + \dim \text{Image}(T) = n.$$

- Section 3.4: Definition of coordinates. Given a set of basis, find the coordinates of a vector; Given a linear transformation and a set of basis, find the matrix of the transformation with respect to the given basis.

Chapter 4

- Section 4.1: Definition of a vector space. You are not required to determine whether a set is a vector space.
- Section 4.1: Definition of subspace in a vector space. Given a subset in a vector space, determine whether it is a subspace.
- Section 4.1: Linear independence/dependence, linear combinations, span, basis, dimension and coordinates.