

### Reduced row-echelon form in plain english:

Here are the three criteria for reduced row-echelon form matrices written in plain(er) english than the textbook. A matrix  $A$  is in reduced row echelon form (rref) if

- (1) Reading from left to right, the first non-zero entry in each row is a one. It's OK for a row to be all zeros, but if it's not then the first non-zero number is 1. Such a one is called a **leading** one. The column it's in is called a **pivot** column.
- (2) Each leading 1 is the only non-zero entry in its *column*. If you hit a leading one, look in its column above and below it; everything should be zero
- (3) The columns of the leading ones should go in numerical order. If the first leading one (the one in the first row) is in the second column, then the next one has to be in at least the third column, and so on. Furthermore, all the rows of zeros come at the bottom.

Here are some non-rref matrices. What property do they fail to exhibit? Answer at bottom of the page. (Cover it up!)

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (0.1)$$

Ans: from left to right, 2, 1, 3, 3.