PROBLEM FOR MATH 201, DUE FRIDAY, OCTOBER 3RD

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Problem 1: Consider the vectors

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad \text{in } \mathbb{R}^3.$$

Prove that

$$\operatorname{span}\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right) = \operatorname{span}\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right).$$

By arguing along the lines discussed in class. Namely, recall that given two sets X and Y in $\mathbb{R}^n$, to prove that $X = Y$, we argued separately that $X \subset Y$ ("X is contained in Y") and that $Y \subset X$ ("Y is contained in X").

Problem 2: Show that the vectors

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

are linearly dependent by using the definition. That is, find three scalars c_1, c_2, c_3, not all equal to zero, such that

$$c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0,$$

On the other hand, prove that

$$\operatorname{span}\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right) \neq \operatorname{span}\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}\right), \tag{0.1}$$

by finding a vector in the set on the left hand side that is not contained in the set on the right hand side. Finally, draw a diagram of the two spans in (0.1).

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