Some questions form Chapters 1, 2 and 3

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More than one answer is possible here

A span is

1. a basis for a vector space.
2. a finite set of vectors.
3. an infinite set of vectors.
4. a linear subspace.
5. a set of all linear combinations of a set of vectors.

More than one answer is possible here

For $T : \mathbb{R}^m \to \mathbb{R}^n$, $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$,

1. $\mathrm{im}(T) \subset \mathbb{R}^m$.
2. $\mathrm{im}(T) \subset \mathbb{R}^n$.
3. $\ker(T) \subset \mathbb{R}^m$.
4. $\ker(T) \subset \mathbb{R}^n$.

More than one answer is possible here

A basis of an n-dimensional vector space V is

1. any finite set of vectors in V.
2. an infinite set of vectors in V.
3. The span of a set of vectors in V.
4. any linearly independent set of vectors in V.
5. any linearly independent set of vectors in V that span V.

More than one answer is possible here

For $T : \mathbb{R}^m \to \mathbb{R}^n$, $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$, $\mathrm{im}(A) =$

1. all solutions to $\mathbf{A}\mathbf{x} = \mathbf{b}$, $\forall \mathbf{b} \in \mathbb{R}^n$.
2. all solutions to $\mathbf{A}\mathbf{x} = \mathbf{0}$.
3. all $\mathbf{b} \in \mathbb{R}^n$ where $\mathbf{A}\mathbf{x} = \mathbf{b}$ is consistent.
4. all points in $\mathbb{R}^m$ mapped to a particular $\mathbf{b} \in \mathbb{R}^n$.

True or False

1. The column vectors of any 5×4 matrix must be linearly dependent.
2. If $\mathbf{A}$ is an invertible $n \times n$ matrix, then the kernels of $\mathbf{A}$ and $\mathbf{A}^{-1}$ must be equal.
3. If the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots \mathbf{v}_n$ span $\mathbb{R}^4$, then n must be equal to 4.
4. The image of a 3×4 matrix is a subspace of $\mathbb{R}^4$.
5. If $\mathbf{A}^2 = \mathbf{I}_n$, then $\mathbf{A}$ must be invertible.
6. The function $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x - y \\ y - x \end{bmatrix}$ is a linear transformation.
7. if $\mathbf{AB} = \mathbf{I}_n$ for two matrices $\mathbf{A}$ and $\mathbf{B}$, the $\mathbf{B}$ must be the inverse of $\mathbf{A}$.
8. There exists a 3×4 matrix with rank 4.
9. A linear system with fewer unknowns than equations must have either an infinite number of solutions or no solutions.
10. A matrix $\mathbf{E}$ is in reduced-row echelon form. If we remove any single row, the resulting matrix will still be in reduced-row echelon form.