Name ....

SAMPLE MIDTERM 2 *100pts.*

- **There are 6 pages in the exam including this page.**
- **Write all your answers clearly. You have to show work to get points for your answers.**
- **You can write on both sides of the paper. Indicate that the answer follows on the back of the page.**
- **Use of Calculators is *not* allowed during the exam.**

(1) ..../22

(2) ..../22

(3) ..../20

(4) ..../36

Total ....../100

(1) *22pts.* Let $\mathcal{P}_2$ denote the set of all polynomials of degree less than or equal to 2. Let $T : \mathcal{P}_2 \to \mathcal{P}_2$ be a linear transformation defined as $T(f(t)) = f(t-1)$, where $f(t)$ is a polynomial of degree less than or equal to 2.

(a) Find the matrix of this transformation with respect to the basis $\{1, t, t^2\}$ of $\mathcal{P}_2$. Show work.

(b) Evaluate the determinant of the matrix you found in part (a). Show work.

(2) *22 pts.* Let $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ be a basis of the subspace W of $\mathbb{R}^3$.

(a) Find a orthonormal basis of W. Show work.

(b) Find the orthogonal projection of the vector $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ onto W. Show work.

(3) *20pts.* Let $\mathbb{C}$ denote the set of complex numbers $\{a + bi : a, b \in \mathbb{R}\}$. Then both $\mathcal{B}_1 = \{1, i\}$ and $\mathcal{B}_2 = \{1 + i, 1 - i\}$ are bases for $\mathbb{C}$.

(a) What is the matrix that transforms a vector in $\mathcal{B}_1$ coordinates into a matrix in $\mathcal{B}_2$-coordinates? Show work.

(b) Write down the element $4 + 2i$ in $\mathcal{B}_2$-coordinates. Show work.

(4) *16pts.* Give short answers to the following.

(a) Let $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -3$. Compute $\det \begin{bmatrix} a+d & b+e & c+f \\ 2g & 2h & 2i \\ d & e & f \end{bmatrix}$.
Give reasons to support your answer.

(b) Let A be a 2×2 matrix such that $\det A = -1$ then A is orthogonal. State true or false with justification.

(c) Let u, v, w be vectors in $\mathbb{R}^n$. Let w be orthogonal to both u and v. Then $u + v$ is orthogonal to $3w$. State true or false with justification.

-END-