Name ....

PRACTICE EXAM 2 *40pts.*

- **There are 6 pages in the exam including this page.**
- **Write all your answers clearly. You have to show work to get points for your answers.**
- **You can write on both sides of the paper. Indicate that the answer follows on the back of the page.**
- **Use of Calculators is *not* allowed during the exam.**

(1) ..../8

(2) ..../8

(3) ..../8

(4) ..../16

Total ..../40

(1) *8pts.* Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be defined by a matrix

$$A = \begin{bmatrix} 1 & 0 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 3 & 2 & 0 & 1 \\ 1 & -1 & 0 & 4 \end{bmatrix}$$

(a) Find the Kernel of T.

(b) Is T invertible? Why or why not?

(2a) *3pts* When is a set of vectors $\{v_1, \cdots, v_n\}$ in a vector space V said to be a basis of V?

(2b) *5pts.* Let $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 4 & 1 & 2 \\ -1 & 2 & 1 & 1 \\ 2 & 0 & -1 & 0 \end{bmatrix}$ represent a linear transformation from $T : \mathbb{R}^4 \to \mathbb{R}^4$. Find a basis for the $\mathrm{Im}T$.

(3) *8pts.* Check if the following sets are subspaces of $I\!R^3$ and $I\!R^4$ respectively or not. Explain your answers.

(a) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : \begin{array}{c} x - y + z = 0 \\ x + 1 - 2z = 0 \end{array} \right\}$

(b) $V = \left\{ \begin{bmatrix} 0 \\ a + b \\ c \\ c - 5a \end{bmatrix} : a, b, c \in I\!R \right\}.$

(4) *16pts.* Give short answers to the following.

(a) If A and B are 2×2 matrices such that $AB = 0$ then, either $A = 0$ or $B = 0$.

(b) Suppose that A is a 3×3 matrix such that $Ax = x$ for all $x \in \mathbb{R}^3$. Let I_3 be the 3×3 identity matrix. Find $\mathrm{Ker}(A - I_3)$, that is the Kernel of the transformation represented by $A - I_3$.

(c) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation which maps a vector $v \in \mathbb{R}^2$ to its reflection along a line along $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Describe the matrix of this transformation.

(d) Is $\left\{ \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix} \right\}$ a basis of $\mathbb{R}^3$? Why or why not?