November 8, 2009

MATH 110.201
Suggested Review Exercises

Here are some exercises to review before the exam.

- Determine whether or not the following sets span $I\!R^2$.

  (a) $\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix}\}$.

  (b) $\{\begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}\}$

- Give short answers to the following.

  (a) Let $\{v_1, v_2, v_3\}$ span a vector space V and $v_3 \in \text{Span}\{v_1, v_2\}$. Show that $\text{Span}\{v_1, v_2\} = V$.

  (b) If $Ax = 0$ for all $x \in I\!R^3$. Find $\text{Ker}(A)$.

  (c) If A is a 4×6 matrix with $dim\,\text{Ker}(A) = 2$. How many pivots does A have ? Why?

- Let V be a vector space of dimension 4 and W be a subspace of V.

  (a) What are the minimum and maximum dimensions W can have? Why?

  (b) Let $\{w_1, w_2, w_3\}$ span W and $w_4 \in V$ be such that $\{w_1, w_2, w_3, w_4\}$ is linearly independent. What is the dimension of W? State the properties of linearly independent sets you are using.

- Let $T : I\!R^3 \to I\!R^3$ be the linear transformation defined as

$$T(\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}) = \begin{bmatrix} y_1 + y_2 \\ y_1 - y_2 \\ y_3 \end{bmatrix}$$

  (a) Find the Kernel of T.

  (b) Is T invertible? Explain why or why not?

- Is $W = \{\begin{bmatrix} a-b \\ b \\ 0 \end{bmatrix} : a, b \in I\!R\}$ a subspace of $I\!R^3$ (This is the set of vectors in $I\!R^4$ of the form $\begin{bmatrix} a-b \\ b \\ 0 \end{bmatrix}$ for all possible real values of a and b) ?

(a) Show that W is a subspace of $\mathbb{R}^3$.
(b) Find a basis for W.

- State whether the following statements are true or false. If true, explain your answer. If false, give an example for which the statement is false.
  (a) Let W be a subspace of $\mathbb{R}^4$. If $W = \text{Span}\{\vec{w}_1, \cdots, \vec{w}_k\}$ for vectors $\vec{w}_1, \cdots, \vec{w}_k$ in $\mathbb{R}^4$ and the dimension of W is 3 then $k = 3$.
  (b) If A is a orthogonal 3×3 matrix then $\det A > 0$.
- Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined as

$$T\left(\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}\right) = \begin{bmatrix} y_1 + y_2 \\ y_1 - y_3 \\ 0 \end{bmatrix}$$

  (a) Find a basis of the Image of T.
  (b) What is the Rank of the matrix defining the linear transformation T? Explain your answer.
- Let $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a + b + c = 0 \right\}$ (that is, the set of all vectors $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3$ satisfying the equation $a + b + c = 0$).
  (a) Show that W is a subspace of $\mathbb{R}^3$.

  (b) Find a basis of W.
- Let $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ be a basis of a subspace W of $\mathbb{R}^4$.
  (a) Find an orthogonal basis $\{\vec{u}_1, \vec{u}_2\}$ of W. Show work.

  (b) Let $D = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$ and $Q = [\vec{u}_1 \vec{u}_2]$. Find an upper triangular 2×2 matrix R so that $D = QR$. Show work.
- Let $\mathbb{C}$ be the set of all complex numbers, that is, $\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$. Let $T : \mathbb{C} \to \mathbb{C}$ be the linear transformation defined as $T(a+bi) = -b+ai$
  (a) Find the matrix B of the transformation T with respect to the basis $\{1 + i, 1 - i\}$. Show work.

  (b) Find the matrix A of the transformation T with respect to the basis $\{1, i\}$. Show work.
  (c) Find a matrix S such that $A = SBS^{-1}$. Show work.
- What is the dimension of the vector space of all 1×3 matrices $\mathcal{M}_{1\times 3}$? Explain your answer.

- Let $\mathcal{D}$ denote the space of differentiable functions from $I\!R \to I\!R$. Is the function $<,>: \mathcal{D} \times \mathcal{D} \to I\!R$ defined as
$$< f, g >= f(0)g'(0) + f'(0)g(0)$$
an inner product on $\mathcal{D}$?

- Let $V = \text{Span}\{\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}\}$. Find the dimension of V. Explain your answer.

- Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

  (a) Find the eigenvalues of A.
  (b) Find the eigenspaces corresponding to eigenvalues of A.