Merry with matrices

I will record how $D : P_2 \to P_2$ (the linear operation of taking the derivative) is expressed in coordinates as a matrix. Which coordinates? The ones I choose; the ones you choose; the most convenient ones; .... In this case, we're probably talking about the "standard" basis $\mathfrak{A} = \{1, x, x^2\}$. **Forget all other bases of P_2, unless we decide to change the basis.** We look at what D does to the basis vectors, and read that in terms of the basis vectors (i.e., $\mathfrak{A}$-coordinates):

$$D(1) = 0 = 0{\cdot}1 + 0{\cdot}x + 0{\cdot}x^2, \quad D(x) = 1 = 1{\cdot}1 + 0{\cdot}x + 0{\cdot}x^2, \quad D(x^2) = 2x = 0{\cdot}1 + 2{\cdot}x + 0{\cdot}x^2.$$

So, with

$$[1]_{\mathfrak{A}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad [x]_{\mathfrak{A}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad [x^2]_{\mathfrak{A}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$[D(1)]_{\mathfrak{A}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad [D(x)]_{\mathfrak{A}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad [D(x^2)]_{\mathfrak{A}} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}.$$

Putting the above as the columns of our matrix, we obtain:

$$[D]_{\mathfrak{A}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

I'm sure we know how to take derivatives, but the point is to *illustrate* that

$$D(c_0 + c_1 x^2 + c_2 x^2) \text{ is } \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix},$$

with the column vector outcome interpreted as $\mathfrak{A}$-coordinates of a polynomial in P_2. Check that this reproduces the expected outcome.