**Dysmatria**

Today, I displayed a new cognitive disorder, one that I decided to call *dysmatria*. When I wanted to compute $A^T A$, I wrote $AA^T$.

It was a straightforward least squares calculation: Give the best approximate solution of $A\vec{x} = b$, where

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad \text{and} \quad \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}.$$

For this situation, with $\ker A = \{\vec{0}\}$ the answer is $\vec{x}^* = (A^T A)^{-1} A^T \vec{b}$.

This proceeds as follows.

$$A^T A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}.$$

This product is the *necessarily invertible, symmetric* (why?) $2 \times 2$ matrix

$$\begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}.$$

Its inverse is

$$(A^T A)^{-1} = \tfrac{1}{6} \begin{bmatrix} 14 & -6 \\ -6 & 3 \end{bmatrix}.$$

I quit while I'm ahead (assuming that's true). We must multiply the above matrix by the vector $A^T \vec{b}$ to get $\vec{x}^*$ explicitly.