Exam 1 and Conceptual Points

You learned the basic computational skills in Week 1. Thereafter, you are asked to judge whether to calculate, why to calculate, and how to calculate. With this said, here are some conceptual points:

1. You will not do well by rejecting the premises of the subject (linear algebra). You must understand the main conceptual points.

2. Problems #8 and #9 of Exam 1 were about fundamental conceptual points mentioned repeatedly in lecture and in on-line documents (also in the book, and presumably in section). These points concern linearity and the role of a basis. You may know by now how little needed to be written to answer those problems, but we give it anyway:

#8. $(\vec{x} - \vec{x}_1) \in \ker A \Leftrightarrow A(\vec{x} - \vec{x}_1) = 0 \Leftrightarrow A\vec{x} - A\vec{x}_1 = 0 \Leftrightarrow A\vec{x} = \vec{b}$.

#9. *T is determined by its values on a basis:* every vector $\vec{v} \in \mathbb{R}^3$ is written in one and only one way as $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$ (a linear combination of the basis vectors). Then

$$T(\vec{v}) = T(c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3) = c_1T(\vec{v}_1) + T(c_2\vec{v}_2) + T(c_3\vec{v}_3) = c_1\vec{w}_1 + c_2\vec{w}_2 + c_3\vec{w}_3$$

is the one and only thing to do.

3. By specifying a basis of an n-dimensional linear space V, it enables V to commit "identity theft" against $\mathbb{R}^n$. Let $\mathfrak{B} = \{v_1, ..., v_n\}$ be our present choice of a basis of V. By the coordinate mapping T—linear!—v_1 "claims that it is" $\vec{e}_1$, v_2 "claims it is" $\vec{e}_2$, ... , v_n "claims it is" $\vec{e}_n$. Then $w = \Sigma\, k_i v_i$ is *under orders* to "claim it is" $\vec{x} = \Sigma\, k_i \vec{e}_i$ for each choice of scalars $k_1, ..., k_n$ (the column vector with these numbers as entries, in order, is the $\mathfrak{B}$-coordinate vector of w). In effect, every element of V claims to be a column vector, and it is difficult to "detect" the theft because the operations of vector addition and scalar multiplication are respected.

Said precisely, and at the level of our course, $T(v_i) = \vec{e}_i$ defines an isomorphism (linear one-to-one correspondence) from V to $\mathbb{R}^n$. By #9, this determines the entire coordinate mapping T, whose inverse T^{-1} takes $\vec{e}_i$ to v_i for each i. The rest is determined by linearity, e.g., $T^{-1}(\vec{e}_1 + 7\vec{e}_2 - \vec{e}_3) = v_1 + 7v_2 - v_3$. Taking T^{-1} removes the "disguise" (undoes the identity theft).

In the notation from the textbook, T above is $[\ \ ]_{\mathfrak{B}}$.

Once, I met a student for the first time in an informal setting. Upon introducing myself and seeing his reaction, I asked him what he had heard about me. "You make sure that the students learn the concepts." In reality, I can't make sure of anything. That's each student's job. In linear algebra, the concepts are very specific, so you must get yourself to learn them as stated, and use them wherever they apply.