1. For which choice(s) of the constant k is the following matrix invertible?

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}.$$

2. Find bases of (i) the image, and (ii) the kernel of the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 4 \end{bmatrix}.$$

3. (a) Write down the properties defining a linear subspace $V \subset \mathbf{R}^n$.

   (b) Consider the subset $W \subset \mathbf{R}^3$ consisting of those vectors $\vec{w}$ such that $\|\vec{w}\| = 1$. Justify whether or not W is a linear subspace of $\mathbf{R}^3$ and, if so, find a basis for it.

4. Consider the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ with matrix

$$A = \frac{1}{5}\begin{bmatrix} 9 & -8 \\ 2 & 1 \end{bmatrix}.$$

Find the matrix B of T with respect to the basis $\mathfrak{B}$ of $\mathbf{R}^2$ consisting of the vectors $\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. (Note: B is a much simpler matrix than A. Do you recognize it as the matrix of a shear? T is a shear along $\vec{v}_1$.)

5. (a) Find the matrix A of the reflection T_1 along the diagonal $x_1 = x_2$ in $\mathbf{R}^2$. (Hint: Find the columns of A by applying T_1 to the standard basis vectors $\vec{e}_1, \vec{e}_2$.)

   (b) Find the matrix B of the rotation T_2 around the origin of $\mathbf{R}^2$ by an angle of $-\pi/2$ (a quarter-revolution clockwise).

   (c) What transformation is the composition $T_2 \circ T_1$? Find its matrix.

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