NAME:

SECTION # OR TA'S NAME:

LINEAR ALGEBRA – SECOND MIDTERM EXAM – NOVEMBER 28, 2001

Please attempt all the problems and show all your work. Don't hesitate to ask me for clarification on any questions you may have. You may **not** *use any notes, books or calculators.*

1 . (25 points) Find the characteristic polynomial and **all** eigenvalues and eigenvectors of the matrix:

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix} .$$

2 . (25 points) Find the least squares line best fitting the following data :

$$(0,1)\ ,\ (1,2)\ ,\ \text{and}\ (-1,-3)\ .$$

3 . Answer the following questions **true or false** (5 points each):

(i) For any $n \times n$ matrix A, $\det(A) = \det(\mathrm{rref}(A))$.

(ii) $\det \begin{pmatrix} 2 & 1 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 & 2 \\ 5 & 3 & 0 & 1 & 9 \\ 7 & 4 & 0 & 0 & 4 \\ 9 & 5 & 3 & 4 & 1 \end{pmatrix} = 3.$

(iii) For any $m \times n$ matrix A, $\ker(A) = \ker(A^T A)$.

4 . Answer the following (5 points each):

(i) Give an example of an orthogonal 3×3 matrix other than the identity.

(ii) Is it possible to find an example of a skew-symmetric invertible 3×3 matrix? Explain.

5 . (25 points) Consider the vector space of continuous functions on $[0, 1]$ with inner product

$$\langle f, g\rangle = \int_0^1 f(x)g(x)dx \ .$$

Find the orthogonal projection of the function $h(x) = x^2$ onto the subspace spanned by $\{1, x\}$.

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