First Examination

50 minutes. Closed book. No notes. 80 points, 20 per question.
Partial credit may be available, but only if you show your working.
HINT: Many answers can be checked by direct substitution or other methods.
Use only the officially provided blue books.
Begin each of the four questions on a fresh page and number it clearly in the margin.

1. (a) Solve the linear system

$$\left\{\begin{array}{rrrrcl} x & +y & -2z & +t & = & 5 \\ 2x & +y & -3z & -t & = & 0 \\ x & -y & & -t & = & 1 \end{array}\right.$$

(b) Note that the solution in (a) is not unique.
(i) Is there a solution with $x = 75$? If so, use (a) to write one down.
(ii) Is there a solution with $y = 75$? If so, use (a) to write one down.
(iii) Is there a solution with $z = 75$? If so, use (a) to write one down.
(iv) Is there a solution with $t = 75$? If so, use (a) to write one down.

2. (a) Evaluate the determinant (where t is a variable)

$$d = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ t & t^2 & t^3 \end{vmatrix}$$

(b) For what values of t is $d = 0$?
(c) *Without using* (a), explain why the values of t listed in (b) make $d = 0$.

3. For each of the statements below, state whether it is true or false, and give a reason. All matrices appearing are understood to be $n \times n$ matrices.
(a) $(A + B)^2 = A^2 + 2AB + B^2$;
(b) $(AB)^{-1} = A^{-1}B^{-1}$, assuming that A and B are invertible;
(c) $5(A + B) = 5A + 5B$;
(d) If $A^2 = I$, then $A = \pm I$;
(e) If $AB = I$, then $B = A^{-1}$.

4. By any method, compute the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 2 & -3 & 1 \end{bmatrix}$$