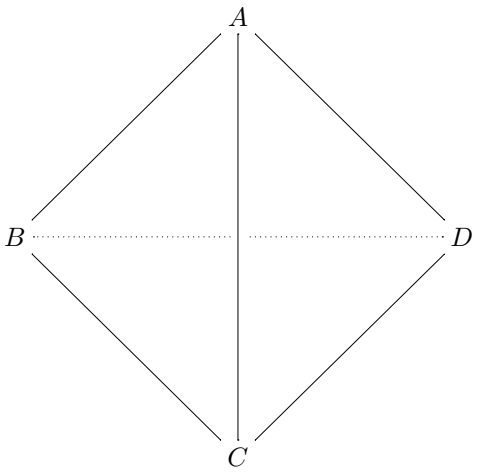
Special Geometry Exam, Fall 2008, W. Stephen Wilson. Mathematics Department, Johns Hopkins University

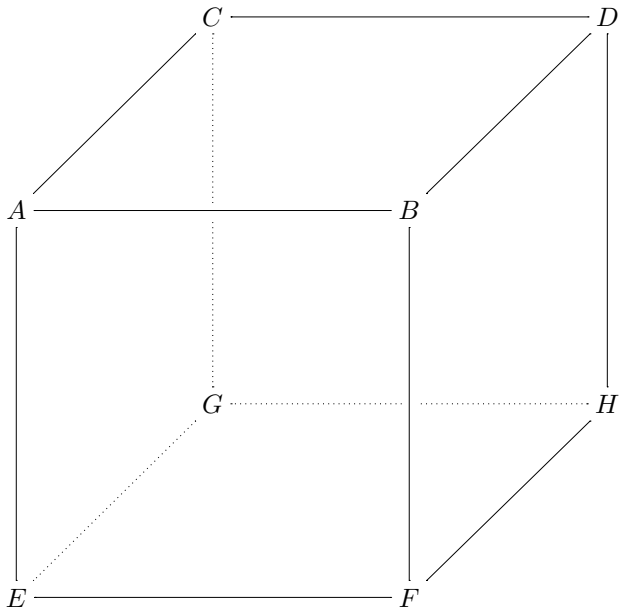
I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name print and sign: _____ Date: _____

NO CALCULATORS, NO PAPERS, SHOW WORK. Put your final answer in the box provided.

Consider the regular tetrahedron and the cube, both with edges of length one unit.





There are three (3) distinct planes that divide the solid regular tetrahedron into two identical pieces and intersect the tetrahedron in a square. Remember, the edges of the tetrahedron are of one unit length.

1. What is the area of the square mentioned above?

2. What is the intersection of two (2) of those planes mentioned above and the tetrahedron? If it is an area, give the area; it if is a line segment, give the length; if it is points, say how many; if there are no points, just put zero (0).

3. What is the intersection of all three (3) of those planes mentioned above and the tetrahedron? If it is an area, give the area; it if is a line segment, give the length; if it is points, say how many; if there are no points, just put zero (0).

4. Find the area of the rectangle formed by the intersection of a plane with the cube when the plane contains four (4) vertices of the cube but it is not a face of the cube.

5. A rhombus is a parallelogram with all sides the same length. There is a plane that intersects the cube, going through exactly two (2) vertices, such that the intersection is a rhombus. What is the length of the side of the rhombus?

6. What is the height of the above rhombus?

7. There are planes that intersect the cube in a regular hexagon. What is the length of a side of such a hexagon?

8. Find a plane that goes through exactly three (3) vertices. On one side of the plane there are four (4) vertices and on the other side there is one (1) vertex. How far is the plane from the single vertex?

9. The plane in the last problem divides the cube into a big piece and a little piece. What is the volume of the big piece?

10. Using the letters that label the cube on the first page, find a non-trivial way to pick up the cube and set it down looking the same (i.e. a transformation), which, when iterated three (3) times puts the cube back where it was. For example, A to C to D to B to A and E to G to H to F to E gives an example of such a transformation that takes four (4) iterations to get back where you started. Note that it is enough to say where one edge goes because the entire rest of the cube must follow, so, for example, the above example of order 4 is completely described by saying that AC goes to CD. Make your own box.