problems	$1-5$	$6-9$	$10-14$	$15-19$	$20-21$	$22-23$	$24-25$	26	*total*
points	20	20	21	20	14	18	20	30	163
scores									

Final Exam, December 9, Calculus II (107), Fall, 2015, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name (signature): ______________________________ Date: ______________

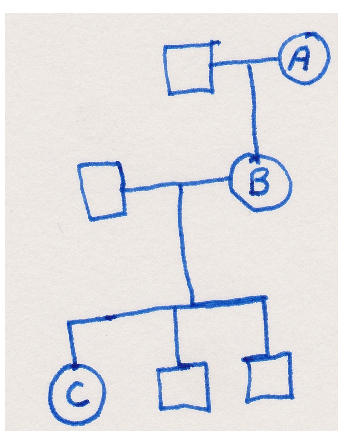
Name (print): ______________________________

TA Name and section: ______________________

NO CALCULATORS, NO PAPERS, SHOW WORK, FILL IN ALL ANSWERS IN ANSWER SHEETS.

We often use the function $F(x) = \int_{-\infty}^{x} \frac{e^{\frac{-u^2}{2}}}{\sqrt{2\pi}} du$ in the solution of problems, where you mainly have to find x. We always use the histogram approach when using the normal distribution. Since we are not using tables, you can use negative values of x in $F(x)$ in your answers.

We assume that $\int_{-2}^{2} \frac{e^{\frac{-u^2}{2}}}{\sqrt{2\pi}} du = F(2) - F(-2) = 2F(2) - 1 = .95$ and $\int_{-3}^{3} \frac{e^{\frac{-u^2}{2}}}{\sqrt{2\pi}} du = .99$ and when you see .95 or .99 show up, it means you should use this. You need $F(1.28) = .9$ for some problems.



1.

(a)(1 point) (b)(1 point) (c)(2 point)

2.

(a)(2 point) (b)(3 point)

3.

(a)(1 point) (b)(1 point) (c)(1 point)

4.

(a)(2 point) (b)(3 point)

5.

(a)(1 point) (b)(1 point) (c)(1 point)

6.

(a)(1 point) (b)(1 point) (c)(1 point)

7.

(a)(1 point) (b)(3 point)

8.

(a)(1 point) (b)(2 point)

9.

(a)(1 point) (b)(2 point)

(c)(3 point) (d)(4 point)

10.

(a)(1 point)

(b)(1 point)

(c)(2 point)

11.

(a)(2 point)

(b)(2 point)

(c)(2 point)

12.

(3 point)

13. (4 point)

14. (4 point)

15. (3 point)

16. (3 point)

17. (3 point)

18. (a) (2 point)

(b) (2 point)

(c) (2 point)

(d) (2 point)

19. (3 point)

20. (a) (2 point) (b) (2 point)

21. (a) (3 point)

(b) (3 point)

(c) (2 point)

(d) (2 point)

22.

(a)(2 point) (b)(2 point) (c)(2 point)

23. (a) (4 point)

(b) (2 point)

(c) (2 point)

(d) (2 point)

(e) (2 point)

24.

(a) (3 point)

(b) (3 point)

(c) (3 point)

25.

(a) (6 point)

(b) (2 point)

(c) (3 point)

26.

(a) (6 point)

(b) (8 point)

(c) (4 point)

(d) (3 point)

(e) (3 point)

(f) (3 point)

(g) (3 point)

1. (4 points) There are 15 classes of 20 students each (300 different students).

How many distinct committees could you make by
(a) (1 point) taking one student from each class.
(b) (1 point) taking 20 students from each of 14 classes.
(c) (2 point) taking two students from each class.

2. (5 points) There are 15 classes of 20 students each (300 different students).

How many distinct committees could you make by
(a) (2 point) taking three students from each of 10 classes.
(b) (3 point) taking 2 students from each of 5 classes and 3 students from 7 different classes.

3. (3 points) Our sample space Ω is 5 balls with the numbers 1-5 on them. We have a random variable that takes each ball to the number written on it, i.e., if we let $B(a)$ be the ball with a on it, then $X(B(a)) = a$. We have probabilities, $P(B(1)) = .1 = P(B(2))$, $P(B(3)) = .3 = P(B(4))$, and $P(B(5)) = .2$.

(a) (1 point) What is $E(X)$?
(b) (1 point) Pull out two balls. What is the probability that the second ball is $B(4)$?
(c) (1 point) What is the probability of picking a ball with a 1, 3 or 5 on it if you pick just one?

4. (5 points) Same as on previous page. Our sample space Ω is 5 balls with the numbers 1-5 on them. We have a random variable that takes each ball to the number written on it, i.e., if we let $B(a)$ be the ball with a on it, then $X(B(a)) = a$. We have probabilities, $P(B(1)) = .1 = P(B(2))$, $P(B(3)) = .3 = P(B(4))$, and $P(B(5)) = .2$.

(a) (2 point) Pull out two balls. What is the probability that the second ball is $B(4)$, if the first ball was $B(1)$?
(b) (3 point) Pull out two balls. What is the probability that the sum of the numbers on the two balls is 5? Write as a single fraction in reduced form (no common divisors in numerator and denominator).

5. (3 points total, 1 point each)

We have 4 red balls in an urn with 5 green balls. Pick two with no replacement.
(a) What is the probability that both are red?
(b) What is the probability that both are green?
(c) What is the probability that there is one red and one green?

6. (3 points total, 1 point each)

We have 4 red balls in an urn with 5 green balls.
Pick two with replacement.
(a) What is the probability that both are red?
(b) What is the probability that both are green?
(c) What is the probability that there is one red and one green?

7. (4 points total)

We have a Monty Hall type problem with 5 doors. There is one good prize behind one of the doors. You pick one but it doesn't get opened. Monty Hall opens two doors without the good prize. You then have to choose to keep the door you have or to switch to another unopened door.
(a) (1 point) What is the probability you get the prize if you don't switch?
(b) (3 point) What is the probability you get the prize if you do switch?

8. (3 points total)

Same as previous page. We have a Monty Hall type problem with 5 doors. There is one good prize behind one of the doors. You pick one but it doesn't get opened. Monty Hall opens two doors without the good prize. You then have to choose to keep the door you have or to switch to another unopened door.
(a) (1 point) What is the probability you don't get the prize if you don't switch?
(b) (2 point) What is the probability you don't get the prize if you do switch?

9. (10 points total)

Using the hemophilia pedigree on the first page of the exam, assume that A is a carrier. Next page blank for more room.
(a) (1 point) What is the probability that B is a carrier?
(b) (2 point) What is the probability that C is a carrier?
(c) (3 point) What is the probability that B is a carrier given that she has those 2 sons who are not hemophiliacs?
(d) (4 point) What is the probability that C is a carrier given that she has those 2 brothers who are not hemophiliacs?

This page blank for computations for previous page's problem.

10. (4 points total)

We have our usual independent random variables X_k coming from flipping an unfair coin such that the probability of getting a head on the k-th flip is p, i.e. $E(X_k) = p$.
(a) (1 point) For our usual S_n, what is the probability of getting k heads?
(b) (1 point) Using the Poisson approximation, what is the probability that $S_n = k$?
(c) (2 point) Find b and a for the probability that $S_n = k$ using the normal distribution, $\int_a^b \frac{e^{\frac{-u^2}{2}}}{\sqrt{2\pi}} du$.

11. (6 points total)

The probability of getting an A in a class is $\frac{1}{10}$. There are 300 students. Use histograms with the normal distribution and write the answer in terms of the function $F(x)$.
(a) (2 point) Find the probability that 25 or fewer get an A.($\approx 19\%$)
(b) (2 point) Find the probability that 35 or more get an A.($\approx 19\%$)
(c) (2 point) Find the probability that between 27 and 33 get an A.($\approx 50\%$)

12. (3 points total) State what is meant by **convergence in probability.**

13. (4 points)

We have an unfair coin with a probability of getting a head equal to $\frac{1}{4}$. How many times must we flip the coin to be 95% sure that our usual $|\bar{X}_n - \frac{1}{4}| \leq .01$? The answer is a nice round number.

14. (4 points)

Assuming the coin the the previous problem. If we flip it 30,000 times, what is the best estimate c that we can get to be sure with probability .95 that $|\bar{X}_n - \frac{1}{4}| \leq c$? This is a nice round number to 3 decimal places.

The next 5 problems on probability are from the last homework where I promised some would be on the exam. These should be freebies for you.

15. (3 points) Find the points of inflexion for $\frac{e^{\frac{-u^2}{2}}}{\sqrt{2\pi}}$.

16. (3 points)

Assume normal distribution with mean μ and standard deviation σ. What is the probability of falling within $(-\infty, \mu + 3\sigma)$?

17. (3 points)

Suppose X is normally distributed with mean -1 and standard deviation 1. What is the probability $P(-1.5 < X < 2.5)$. (Answer in terms of $F(x)$.)

18. (8 points)

The total maximum score on a calculus exam was 100 points. The mean score was 74 and the standard deviation was 11. Assume that the scores are normally distributed. Use histograms on (a) and (b) and write answers in terms of $F(x)$. Do not use histograms on (c) and (d) and round answers to nearest whole score. Next page blank for more work space.
(a) (2 points) Determine the percentage of students scoring 90 or above? ($\approx 8\%$)
(b) (2 points) Determine the percentage of students scoring between 60 and 80 (inclusive). ($\approx 63\%$)
(c) (2 points) Determine the minimum score of the highest 10% of the class.
(d) (2 points) Determine the maximum score of the lowest 2.5% of the class.

Blank page for computations.

19. (3 points)

How often do you have to toss a coin to determine P(heads) within 0.1 of its true value with probability at least .8?

20. (4 points)

Consider $\int_0^\infty e^{-x}\sin(x)dx$.
(a) (2 points) Does this converge or does it diverge?
(b) (2 points) If it converges, say what it converges to. If it diverges, say if it diverges to ∞, $-\infty$, or neither.

21. (10 points total)

Consider the differential equation $\frac{dy}{dx} = ye^{-y}\cos(y)$ where we restrict to $y \in [-\pi, \pi]$.
(a) (3 points) Find the equilibrium points.
(b) (3 points) For each equilibrium point, say if it is stable or unstable.
(c) (2 points) What happens as $x \to \infty$ if at $x = 0$, $y = \frac{\pi}{4}$.
(d) (2 points) What happens as $x \to \infty$ if at $x = 0$, $y = \frac{3\pi}{4}$.

22. (6 points total)

Let $\frac{dy}{dx} = x^4 y$.
(a) (2 points) Find y when $x = 1$ if $y = 0$ when $x = 0$.
(b) (2 points) Find y when $x = 1$ if $y = e$ when $x = 5^{\frac{1}{5}}$.
(c) (2 points) Find the general solution to the differential equation.

23. (12 points total)

(a) (4 points) Find the Leslie matrix for the situation when there are only newborns and one-year olds. Half of all newborns survive to be one-year olds, and each newboard creates one new newboard the next year. Each one-year old creates 12 newborns the next year. Next page blank for computations.
(b) (2 points) What is the ratio of newborns to one-year olds as time goes off to infinity?
(c) (2 points) Find the smallest population that doubles every year if it exists.
(d) (2 points) Find the smallest population that triples every year if it exists.
(e) (2 points) If we start with 25 newborns and no (zero) one-year olds, how many one-year olds will we have in 10 years?

Blank for computations.

24. (9 points total)

Blank page follow if needed. We have 2 lines in 3-space:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \text{ and } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

(a) (3 points) Find a point they meet at.
(b) (3 points) Find an angle they make with each other.
(c) (3 points) Find an equation for a plane of the form $z - z_0 = a(x - x_0) + b(y - y_0)$ that contains both lines and where (x_0, y_0, z_0) is the point the two lines meet at.

Blank.

25. (11 points total)

Consider the system of differential equations:

$\frac{dx}{dt} = x + 12y$
$\frac{dy}{dt} = \frac{x}{2}$

(a) (6 points) Give the general solution to this system.
(b) (2 points) What kind of equilibrium is $(0, 0)$, stable, unstable, or saddle?
(c) (3 points) With initial condition $t = 0$ giving $(x, y) = (6, 1)$, what is (x, y) at $t = 1$?

We work with the simple function $f(x, y) = x\sin(y)$. We consider the region R, a rectangle with four corners $(\pm 1, \pm\frac{3\pi}{2})$. The next two pages are left blank for scratch paper.

26. (30 points total)

(a) (6 points) Find the critical points inside this rectangle and determine if they are local maximal, local minimal, or saddle.
(b) (8 points) In R, including the bounding rectangle, find all the max/min points (and say which is which).
(c) (4 points) Find the level set $f(x, y) = 0$ in R.
(d) (3 points) Find the tangent plane to the graph at $(\frac{1}{2}, \frac{\pi}{2})$.
(e) (3 points) Find the tangent plane to the graph at $(1, 1)$.
(f) (3 points) Find the directional derivative at $(1, 1)$ in the direction $(1, 1)$.
(g) (3 points) Find the tangent line to the graph at $(1, 1)$ in the direction of $(1, 1)$.

Blank.

Blank.