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|-----------------|-----------|------------|-------------|-------------------|
| <i>problems</i> | 1 – 8(16) | 7 – 16(18) | 17 – 20(16) | <i>total</i> (50) |
| <i>scores</i> | | | | |

Exam #2, October 30, Calculus II (107), Fall, 2015, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name (signature): _____ Date: _____

Name (print): _____

TA Name and section: _____

NO CALCULATORS, NO PAPERS, SHOW WORK. (50 points total)

Let $Z = \begin{pmatrix} x \\ y \end{pmatrix}$ with the system of differential equations $\frac{dZ}{dt} = AZ$. In the following problems you will be given an A . You are to say which kind of equilibrium the origin is (of the 6 possible) (1 point) and say why it is that kind (1 point).

1. (2 points) $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

2. (2 points) $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

3. (2 points) $A = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$

4. (2 points) $A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

5. (2 points) $A = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$

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6. (2 points) $A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

7. (2 points) $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

8. (2 points) $A = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$

For the rest of the exam we use the function $f(x, y) = x^2 + y^2 - 2x - 2y + 2$.

9. (2 points) Compute the gradient of f .

10. (2 points) Find all critical points for f .

11. (2 points) Compute the Hessian for f .

12. (3 points) $f(x, y) = x^2 + y^2 - 2x - 2y + 2$. Compute the Hessian at each of the critical points you found and say what kind of critical point each is, i.e. local max, local min, or a saddle point. Say why.

13. (3 points) $f(x, y) = x^2 + y^2 - 2x - 2y + 2$. Find an equation for the tangent plane to the graph at the point given by $(x, y) = (0, 0)$.

14. (2 points) Find the directional derivative of f at $(0, 0)$ in the direction of maximal slope.

15. (2 points) $f(x, y) = x^2 + y^2 - 2x - 2y + 2$. Find the directional derivative of f at $(0, 0)$ in the direction of $(1, -1)$.

16. (2 points) Give an equation for the tangent line to the level curve at $(0, 0)$.

17. (4 points) $f(x, y) = x^2 + y^2 - 2x - 2y + 2$. Find an equation for the tangent line through $f(0, 0)$ in the direction of $(1, -1)$.

18. (4 points) $f(x, y) = x^2 + y^2 - 2x - 2y + 2$. If x and y are functions of t , give the formula for $\frac{df}{dt}$?

19. (4 points) $f(x, y) = x^2 + y^2 - 2x - 2y + 2$. Find the maximum and minimum for the function when restricted to $x^2 + y^2 \leq 4$

20. (4 points) $f(x, y) = x^2 + y^2 - 2x - 2y + 2$. Find the maximum and minimum for the function on the boundary of the unit square with corners $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$. Find the two points and the value of f at them.