problems	1(3)	2 − 6(18)	7 − 11(18)	12 − 15(11)	*total*(50)
scores					

Exam #1, September 28, Calculus II (107), Fall, 2015, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name (signature): ______________________________ Date: ______________

Name (print): ______________________________

TA Name and section: ______________________

NO CALCULATORS, NO PAPERS, SHOW WORK, put answers in BOXES.

1. (3 points) Let $-\infty < s < t < \infty$, evaluate

$$\int_s^t e^{-x} dx$$

or explain why it diverges. Show work, put answers in a box.

2. (4 points) Show work. Put answers in boxes.

Consider the differential equation

$$\frac{dy}{dx} = \frac{y^2 - 4}{2y}.$$

(a) Find all of the equilibrium points for $y > 0$.
(b) State whether each is a stable or unstable equilibrium and explain why.

3. (3 points)

Consider the differential equation

$$\frac{dy}{dx} = \frac{y^2 - 4}{2y}.$$

If $y = f(x)$ is the solution for the differential equation with initial conditions $y = 4$ when $x = 0$, make a rough sketch of the graph.

4. (4 points) Solve the differential equation

$$\frac{dy}{dx} = \frac{y^2 - 4}{2y}.$$

explicitly for the case where $y = 3$ when $x = 0$. Show work and put your answer in a box.

5. (2 points) Show work and put your answer in a box. Consider the differential equation.

$$\frac{dy}{dx} = \frac{y^2 - 4}{2y}$$

If $y = 2$ when $x = 0$, what is y when $x = 2$?

6. (5 points) Write down the Leslie matrix for the situation where each newborn survives to age 1, each 1 year old survives to age 2, there are no 3 year olds, each newborn creates 1 newborn the next year, each 1 year old creates 2 newborns, and each 2 year old creates 1 newborn.

7. (3 points) Show work and put your answer in a box. Solve the simultaneous linear equations:

$$x \begin{pmatrix} 6 \\ 1 \end{pmatrix} + y \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

8. (4 points) Show work and put your answers in a box. Calculate (a) the largest eigenvalue and (b) the smallest eigenvalue for $A = \begin{pmatrix} 1 & 3 \\ \frac{1}{4} & 0 \end{pmatrix}$.

9. (4 points) Show work and put your answers in a box. For $A = \begin{pmatrix} 1 & 3 \\ \frac{1}{4} & 0 \end{pmatrix}$, calculate the eigenvectors for (a) the largest eigenvalue. Choose the one with coordinate 1 for the second coordinate; (b) the smallest eigenvalue. Choose the one with coordinate 1 for the second coordinate.

10. (4 points) For the matrix $A = \begin{pmatrix} 1 & 3 \\ \frac{1}{4} & 0 \end{pmatrix}$, compute $A^k \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

11. (3 points) Show work and put your answers in a box. If $A^k \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} a_k \\ b_k \end{pmatrix}$, compute the limit as k goes to infinity of $\frac{a_k}{b_k}$.

12. (3 points) Show work and put your answers in a box. Interpret the Leslie matrix in terms of newborns and one-year olds: $A = \begin{pmatrix} 1 & 3 \\ \frac{1}{4} & 0 \end{pmatrix}$

13. (3 points) Show work and put your answers in a box. Considering the Leslie matrix used in the last problem. Start with a population of N_0 newborns and N_1 1-year olds with $(N_0, N_1) \neq (0, 0)$, as k goes to infinity, what is the ratio of the newborns to the 1-year olds?

14. (2 points) Show work and put your answers in a box. Using the same Leslie matrix. Find a population (N_0, N_1) such that the total population $N_0 + N_1$ doesn't change from year 0 to year 1.

15. (3 points) Show work and put your answers in a box. Find all solutions to the following equation. If your answer is a line in 3-space, give it in terms of a parametric equation, i.e. $r = r_0 + tu$ where r_0 and u are vectors.

$$\begin{pmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 0 \end{pmatrix}$$