problems	$1-6(34)$	$7-13(34)$	$14-19(34)$	*total*(102)
scores				

Final Exam, December 10, Calculus II (107), Fall, 2014, W. Stephen Wilson

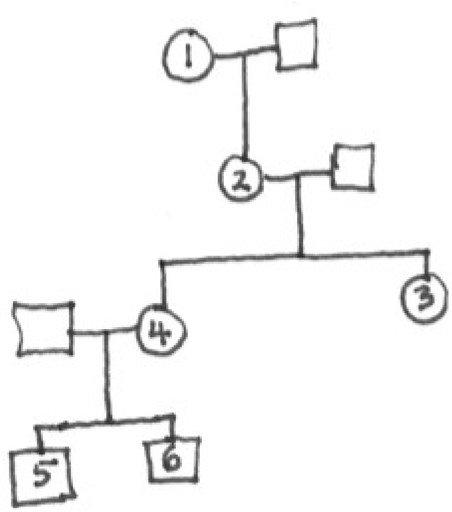
I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name (signature): ______________________________ Date: ______________

Name (print): ______________________________

TA Name and section: ______________________

NO CALCULATORS, NO PAPERS, SHOW WORK. (102 points total)



1. (2 points)

2. (6 points)

at point of type

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3. (8 points)

(a)

(b)

(c)

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4. (a) (4 points) (b) (2 points)

5. (6 points)

(a) (b) (c)

6.

(a) (4 points)

(b) (2 points)

7. (6 points total)

(a) (4 points)

(b) (2 points)

8. (12 points total)

(xy-coordinates, (2 points)) (value, (1 point))

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9. (2 points)

10. (2 points)

11. (8 points)

at point Hessian = of type

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12. (2 points)

13. (2 points)

14. (a) (2 points) (b) (2 points)

15. (a) (2 points) (b) (2 points)

16. (a) (2 points) (b) (2 points) (c) (2 points)

17. (a) (2 points) (b) (2 points) (c) (2 points)

18. (a) (2 points)

(b) (2 points) (c) (2 points)

19. (a) (2 points) (b) (2 points)

(c) (2 points) (d) (2 points)

1. (2 points) Describe the following integral as

(1) convergent
(2) diverges to $-\infty$
(3) diverges to $+\infty$
(4) just diverges
(5) is not improper

$$\int_0^1 \frac{dx}{x}$$

2. (6 points) Let $\frac{dy}{dx} = g(y)$ where the only solutions to $g(y) = 0$ are $y = 1$, 2, and 3. Let $g'(y) = \frac{dg(y)}{dy}$. Assume we know that $g'(1) = 1$, $g'(2) = -1$, and $g'(3) = 1$. List the y that give equilibrium points and state what kind of equilibrium point it is.

3. (8 points total) Consider the differential equation $\frac{dy}{dx} = 4y^2x^3$.

(2 points)	(a) Find the general solution for y.
(2 points)	(b) Find the solution with initial conditions $x = 0$, $y = 1$.
(2 points)	(c) With that initial condition, what is y when $x = 2$?
(2 points)	(d) With that initial condition, find the xy-coordinates for the only equilibrium point and say if it is stable or unstable (or neither).

4. (6 points total) (a) (4 points) Find the Leslie matrix for the situation where half the newborns live to be 1-year olds, there are no 2-year olds, each newborn gives rise to a newborn the next year and each 1-year old gives rise to 4 newborns the next year. (b) (2 points) If we start with 6 newborns and no 1-year olds, how many of each do we have after 1 year?

5. (6 points total) (a) (2 points) Using the (correct) Leslie matrix from the previous problem, what is the ratio of newborns to 1-year olds as the number of years goes off to infinity? (starting with some positive population) (b) (2 points) Find a general formula for the number of 1-year olds after n years if you start at year zero with 6 newborns and zero 1-year olds. (make sure it works with year 0 and 1) (c) (2 points) How many 1-year olds are there after 10 years if we start with 6 newborns and zero 1-year olds in year zero? (Hint: $2^{10} = 1,024$.)

6. (6 points total) Consider the system of differential equations:

$\frac{dx}{dt} = x + 4y$

$\frac{dy}{dt} = \frac{x}{2}$

(a) (4 points) Find the general solution.

(b) (2 points) What kind of equilibrium is $(0, 0)$?

7. (6 points total) (a) (4 points) What is the solution to the system of equations in the previous problem when $x = 6$ and $y = 0$ for $t = 0$. (b) (2 points) For that initial condition, give a formula for y when $t = n$.

Let $f(x, y) = \cos(x)\sin(y)$, with $x^2 + y^2 < \pi^2$.

8. (12 points) There are exactly 4 critical points inside this circle. Find each critical point. Evaluate the function at each point.

Let $f(x, y) = \cos(x)\sin(y)$, with $x^2 + y^2 < \pi^2$.
9. (2 points) Describe the level set $f(x, y) = 0$.

Let $f(x, y) = \cos(x)\sin(y)$, with $x^2 + y^2 < \pi^2$.
10. (2 points) Easy (trick?) question. What is the equation for the tangent plane at the local maximum.

Let $f(x,y) = \cos(x)\sin(y)$, with $x^2 + y^2 < \pi^2$.

11. (8 points) Compute the Hessian at each critical point. State if the point is a local maximum, local minimum, or a saddle point.

12. (2 points) For $F(x, y) = (x^2y^2, xy)$, what is the Jacobi matrix (i.e. derivative) at $(1, 1)$?

13. (2 points) if $\frac{d^2x}{dt^2} = \frac{dx}{dt}$ Solve for $x(t)$. This is a bit non-standard, but you can do it.

14. (4 points) Let $\Omega = \{a, b, c, d\}$ be our sample space with: $P(a) = .1$, $P(b) = .2$, $P(c) = .3$, and $P(d) = .4$. Let $X : \Omega \to \mathbb{R}$ be a discrete random variable given by: $X(a) = 1$, $X(b) = 2$, $X(c) = 3$, and $X(d) = 4$.

(a) (2 points) Evaluate $E(X)$.

(b) (2 points) Compute the variance of X.

15. (4 points) We have 6 balls in a bag, 2 green, 3 red, and 4 black. Pick 3 balls out at random. Give answers below as fractions in reduced form.

(a) (2 points) What is the probability of getting one ball of each color if there is no replacement.

(b) (2 points) What is the probability of getting one ball of each color if there is replacement.

16. (6 points) Using the hemophilia pedigree on the first page of this exam we start with 3 simple problems, assuming that person 1 is a carrier. Give answers below as fractions in reduced form.

(a) (2 points) What is the probability that 2 is a carrier?

(b) (2 points) What is the probability that 3 is a carrier?

(c) (2 points) What is the probability that 4 is a carrier?

17. (6 points total) Using the hemophilia pedigree on the first page of this exam we finish with 3 hard problems, assuming that person 1 is a carrier. Give answers below as fractions in reduced form.

(a) (2 points) What is the probability that 2 is a carrier given the condition that 5 and 6 are not hemophilic?

(b) (2 points) What is the probability that 4 is a carrier given the condition that 5 and 6 are not hemophilic?

(c) (2 points) What is the probability that 3 is a carrier given the condition that 5 and 6 are not hemophilic?

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18. (6 points total) We have our usual independent random variables. The probability of a single event is $\frac{1}{10{,}000}$. Our sample size is $n = 50,000$.

(a) (2 points) Using the binomial distribution, write the precise formula for the probability of exactly 5 events. (probability answer to 5 decimals: .17548)

(b) (2 points) Using the Poisson distribution, write the formula for the probability of exactly 5 events. (probability answer to 5 decimals: .17547)

(c) (2 points) Using the normal distribution, find a, where the following integral would give the estimate for 5 events happening. (probability answer to 5 decimals: .17695)

$$\int_{-a}^{a} \frac{e^{\frac{-u^2}{2}}}{\sqrt{2\pi}} du.$$

19. (8 points total) You suspect that your coin is $\pm .01$ off of being fair. This will mess with your gambling habit. Assume it has probability p of getting a head.

(a) (2 points) If you flip it n times, what is the expected value of S_n (the usual S_n)?

(b) (2 points) If you flip it n times, what is the variance of S_n?

(c) (2 points) If your coin is fair and $n = 100$, what is the standard deviation of S_n?

(d) (2 points) Assume that you have probability of .99 to be within 3 standard deviations using the normal curve. How many times would you have to flip a coin to be 99% sure that your coin was not .01 off from being fair? (Clue. You can calculate an exact number somewhere between 100 and 100,000.)

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