Exam #2, October 29, Calculus II (107), Fall, 2014, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name (signature): _____ Date: _____

Name (print): _____

TA Name and section: _____

NO CALCULATORS, NO PAPERS, SHOW WORK. (51 points total)

1. (4 points) Let $Z = \begin{pmatrix} x \\ y \end{pmatrix}$. We have the general system of linear differential equations given by a matrix A, i.e. $\frac{dZ}{dt} = AZ$. If we have $\lambda_1 \neq \lambda_2$ and $\lambda_1 \neq 0 \neq \lambda_2$ both real, with vectors u_1 and u_2 with $Au_i = \lambda_i u_i$, what is the general solution to the differential equation.

2. (4 points) Find the general solution for the system of differential equations: $\frac{dx}{dt} = y$, $\frac{dy}{dt} = x$.

3. (2 points) Using the same system of differential equations as in the last problem, find $x(1)$ and $y(1)$ when $x(0) = 1 = y(0)$.

4. (2 points) Consider $\frac{d^2x}{dt^2} = x$ Solve for $x(t)$.

5. (2 points) For the equation in the previous problem, if x and $\frac{dx}{dt}$ are 1 at $t = 0$, what is $x(1)$?

For the next part of the exam we use the function $f(x, y) = x^2y + x^2y^2 - xy$.

6. (2 points) Compute the gradient of f.

7. (4 points) There are exactly 2 critical points for this function. (The problem would be harder if I didn't tell you that.) Find them.

8. (4 points) Compute the Hessian for f.

9. (6 points) Compute the Hessian at each of the two critical points. Say what kind of critical point each is, i.e. local max, local min, or a saddle point.

10. (5 points) Find an equation for the tangent plane to the graph at the point given by $(x, y) = (1, 1)$.

11. (2 points) Find the directional derivative of f at $(1, 1)$ in the direction of maximal slope. (Keep in mind that derivatives are numbers.)

12. (4 points) Find an equation for the tangent line through $f(1,1)$ in the direction of maximal slope.

13. (2 points) If x and y are functions of t and $\frac{dx}{dt} = 5$ and $\frac{dy}{dt} = 6$ at $(x, y) = (1, 1)$, what is $\frac{df}{dt}$?

14. (2 points) Give an equation for the tangent line to the level curve at $(1, 1)$.

15. (6 points) Find the maximum and minimum for the function on the boundary of the unit square with corners $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$. Find the two points and the value of f at them.