Exam #1, September 29, Calculus II (107), Fall, 2014, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name (signature): ________________________________ Date: ______________

Name (print): ________________________________

TA Name and section: ________________________

NO CALCULATORS, NO PAPERS, SHOW WORK. (50 points total)

1. (2 points)

$$\int_0^2 \frac{dx}{(x-1)^3}$$

Choose one (show work):

(a) diverges to infinity
(b) just diverges
(c) converges to 0
(d) none of the above

2. (4 points)

(a) Find the general solution (i.e. y as a function of x) for the differential equation: $\frac{dy}{dx} = axy^2$

(b) Find the specific solution to the above equation with the initial condition that $y = 1$ when $x = 0$.

3. (6 points) Consider the differential equation: $\frac{dy}{dx} = (y-1)(y-2)^2(y-3)$.

Find the 3 equilibrium points and state, in each case, whether it is stable, unstable, or a cross between the two. (show work)

4. (8 points) Consider the differential equation: $\frac{dy}{dx} = (y-1)(y-2)^2(y-3)$.

Make a rough sketch of the solution of the differential equation for the 4 initial conditions when $x = 0$: $y = 0, y = 3/2, y = 5/2, y = 4$. Include the equilibrium points in your sketch.

5. (5 points) Write down the Leslie matrix for the situation where 75% of newborns survive to age 1, 25% of 1 year olds survive to age 2, there are no 3 year olds, each newborn creates 3 newborns the next year, each 1 year old creates 2 newborns, and each 2 year old creates 1 newborn.

6. (4 points) Solve the simultaneous linear equations:

$$x \begin{pmatrix} 3 \\ 1 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

7. (3 points) Interpret the Leslie matrix: $A = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$

8. (4 points) Calculate (a) the largest eigenvalue and (b) the smallest eigenvalue for $A = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$.

9. (4 points) For $A = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$, calculate the eigenvectors for (a) the largest eigenvalue. Choose the one with coordinate 1 for the second coordinate; (b) the smallest eigenvalue. Choose the one with coordinate 1 for the second coordinate.

10. (4 points) For the matrix $A = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$, compute $A^k \begin{pmatrix} 2 \\ 2 \end{pmatrix}$.

11. (3 points) Considering the Leslie matrix used in the last few problems. Start with a population of two (2) newborns and two (2) 1-year olds, as k goes to infinity, what is the ratio of the newborns to the 1-year olds?

12. (3 points) Considering the Leslie matrix used in the last few problems. Start with a completely arbitrary non-zero population. As k goes to infinity, what is the ratio of the newborns to the 1-year olds.