

Math 107: Calculus II, Spring 2006: Midterm Exam II
Tuesday April 11, 2005

Give your **name** and at least one piece of information about your section:

Name:

Section day/time:

Section number (1-6):

TA (Tasky/Baber/Kramer):

1. There are five questions. Each is worth 20 points.
2. **Do not open your booklet until told to begin.** The exam will be 50 minutes long.
3. You may **not** use calculators, books, notes or any other paper. Write all your answers on this booklet. Additional paper is available if required.
4. **You must show all your working and explain your answers clearly to obtain full credit!**
5. **Read the questions carefully!** Some questions only require an answer, others require particular explanations. If in doubt, write more!

1. Calculate each of the following partial derivatives:

(a) if $f(x, y) = \cos(xy)$, find $\frac{\partial^2 f}{\partial x \partial y}$;

(b) if $f(x, y) = x^2y + x^3y^3 + 1$, find $\frac{\partial^2 f}{\partial x^2}$;

(c) if $f(x, y) = (x + y)^{-1/2}$, find $\frac{\partial^2 f}{\partial x \partial y}$.

2. Let f be the function of two variables given by

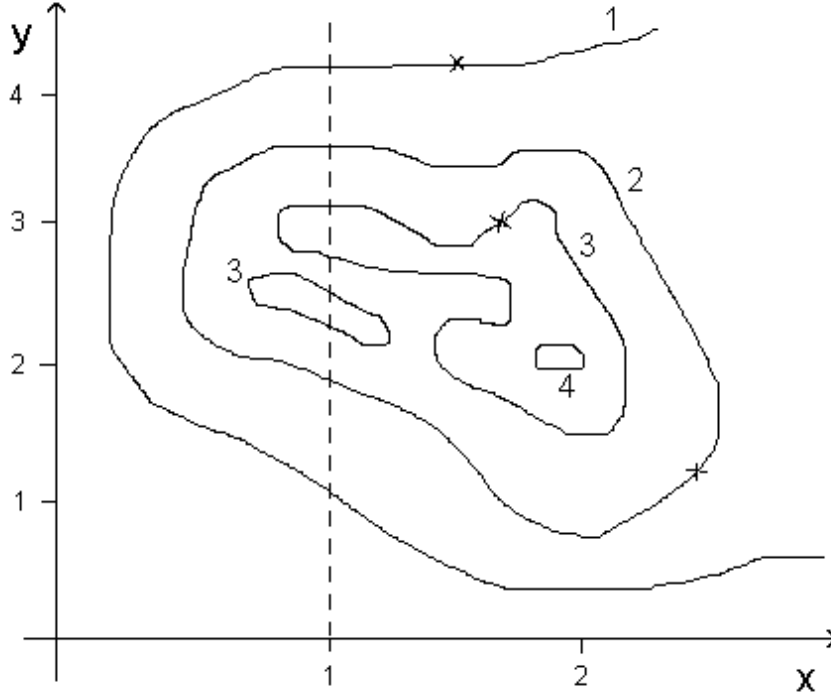
$$f(x, y) = xy(x - y)$$

- (a) Calculate the gradient vector ∇f and evaluate at the point $(2, 1)$.
- (b) What is the directional derivative of f at $(2, 1)$ in the direction of the vector $(1, 1)$?
- (c) In what direction is the directional derivative of f at $(2, 1)$ the least (i.e. the most negative)?

3. Find the linear approximation to the function $f(x, y) = e^{2x} \cos 3y$ at the point $(0, 0)$. Use your approximation to get an estimate of the value of $f(0.1, 0.1)$.

4. Find the critical point of the function $f(x, y) = e^{xy}$. Is this critical point a local max, a local min, or a saddle? (Show your work.)

5. The following diagram displays some of the level curves of a function $f(x, y)$ of two variables. The number labelling a curve 1, 2, 3 or 4 denotes the value of the function f along that curve.



- (a) Sketch the $x = 1$ cross-section through the graph of the function f . (Your graph should be a 2-dimensional graph of z against y , with z giving the value of the function at a particular y . Label the axes of your graph as fully as possible.)
- (b) At each of the three points marked with a cross, draw an arrow that represents the direction of the gradient vector for f at that point. (You should draw the arrows directly on the above diagram.)
- (c) Based on the information in the picture, at roughly what point (x, y) would you expect the global maximum of the function f to be?