

110.107 CALCULUS II
FIRST MIDTERM EXAM
8 October 2001

1. Suppose the amount of phosphorus in a lake at time t , denoted by $P(t)$, follows the equation

$$\frac{dP}{dt} = 3t + 1 \text{ with } P(0) = 0.$$

Find the amount of phosphorous at time $t = 10$.

2. Suppose that a fish population evolves according to the logistic equation, and that a fixed number of fish per unit time are removed. That is,

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right) - H$$

Assuming that $r = 1$ and $K = 100$, find all possible equilibria and discuss their stability when $H = 24$.

3. Let the function $f(x)$ be defined as

$$f(x) = \begin{cases} 3e^{-3x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Show that $f(x)$ is a density function.

4. With X distributed according to $f(x)$ in the previous problem, find the mean of X .
5. Find the cosine of the angle between the two vectors in the plane

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

6. Suppose a quantitative characteristic is normally distributed with a mean $\mu = 20$ and a standard deviation $\sigma = 3$. Find an interval centered at the mean such that 72 % of the population falls into this interval