

Math 107, Spring 2008: Midterm III Practice Exam

On this practice exam, you should use the table in the back of the textbook to find any probabilities you need for the standard normal distribution. On the actual exam, I will give you a list of probabilities that you can use.

The following set of instructions will appear on the exam:

1. There are five questions. Each is worth 20 points.
2. **Do not open your booklet until told to begin.** The exam will be 50 minutes long.
3. You may **not** use calculators, books, notes or any other paper. Write all your answers on this booklet. Additional paper is available if required.
4. **You must show all your working and explain your answers clearly to obtain full credit!**
5. **Read the questions carefully!** Some questions only require an answer, others require particular explanations. If in doubt, write more!

Questions

1. A bag contains two red balls and three blue balls. I perform the following experiment. I pick one ball at random from the bag, and note its color. I then pick a second ball from the bag **without** replacing the first, and note its color.
 - (a) List the possible outcomes of this experiment and find the probability of each outcome.
 - (b) Let A stand for the event “second ball is red” and let B stand for the event “the two balls are the same color”.
 - i. Find $P(A)$.
 - ii. Find $P(B)$.
 - iii. Find $P(A \text{ and } B)$.
 - (c) Are the events A and B independent? Explain how you know.
2. The discrete random variable X has the following distribution:
 - $P(X = 1) = 3/5$
 - $P(X = 2) = 0$
 - $P(X = 3) = 1/5$
 - $P(X = 4) = 1/5$

- (a) Find the expectation and variance of X .
 - (b) Suppose we took 40 independent measurements of X . Find an estimate for the probability that the average of those measurements is greater than 2.4?
3. The continuous random variable X has cumulative distribution function (cdf) given by

$$F(x) = \begin{cases} 0 & \text{if } x < 0; \\ (3x^2 - x^3)/4 & \text{if } 0 \leq x \leq 2; \\ 1 & \text{if } x > 2. \end{cases}$$

- (a) Find the probability density function for X .
 - (b) Find $P(1 \leq X \leq 2)$.
 - (c) Find the expectation and variance of X .
4. (a) Suppose X has a normal distribution with mean 3 and standard deviation 4. Find:
- i. $P(X \leq 5)$
 - ii. $P(X \geq -2)$
- (b) Suppose Y is binomially distributed with 5 repetitions and probability of success $1/3$. Find:
- i. $P(Y = 5)$.
 - ii. $P(Y \geq 1)$.
5. The probability that a Democrat will vote for Hillary Clinton in the presidential primary is 0.5. The probability that they will vote for Barack Obama is 0.5. Suppose that 40,000 Democrats vote.
- (a) Explain how you would use the binomial distribution to find the probability that more than 20,200 people vote for Clinton. (You do not have to make the calculations, but you should describe exactly what calculations you would make.)
 - (b) Use the normal distribution to find an estimate for the probability from part (a). (You do not need to use the histogram correction on this question since it makes so little difference for such a large number of voters.)