**Math 107: Calculus II, Spring 2008: Midterm II Practice Exam 2**
(the actual exam from Fall 2006)

The following instructions will appear on the exam:

1. There are five questions. Each is worth 20 points.
2. **Do not open your booklet until told to begin.** The exam will be 50 minutes long.
3. You may **not** use calculators, books, notes or any other paper. Write all your answers on this booklet. Additional paper is available if required.
4. **You must show all your working and explain your answers clearly to obtain full credit!**
5. **Read the questions carefully!** Some questions only require an answer, others require particular explanations. If in doubt, write more!

## Questions

1. Let $f$ be the function of two variables given by

$$f(x, y) = x^2 - xy + xy^2.$$

   (a) Find the partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

   (b) Draw the $y = 1$ cross-section through the graph of the function $f$. (Hint: this should be a graph of $z$ against $x$.)

   (c) Draw a tangent line to your graph from part (b) whose slope is given by the value of the partial derivative $\dfrac{\partial f}{\partial x}(2, 1)$. (Make sure you label the point at which the tangent line touches the graph.)

   (d) The function $f$ has a saddle point at $(0, 1)$. What does this tell you about the eigenvalues of the Hessian matrix $Hf(0, 1)$? (You may assume that neither eigenvalue is equal to zero.)

2. The function $f(x, y) = x^3 + y^2 - 3x - 2y$ has two critical points. Find these critical points and classify each of them as a local maximum, local minimum or saddle point. (Show all your work.)

3. Let $f$ be the function of two variables given by

$$f(x, y) = e^{x-2y}$$

(a) Find the directional derivative of $f$ at the point $(4, 2)$ in the direction $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(b) In which direction is the directional derivative of $f$ at $(4, 2)$ a maximum? (Your answer should be a vector that points in the appropriate direction.)

(c) What is the size of the directional derivative of $f$ at $(4, 2)$ in the direction you found in part (b)?

(d) Find a direction in which the directional derivative of $f$ at $(4, 2)$ is equal to zero.

4. Let $f$ be the function of two variables given by

$$f(x, y) = y + x^2 - 2$$

(a) Draw a graph that displays the $c$-level curves of $f$ for $c = 0, 2$. (Make sure your graph includes the following ranges on the axes: $-2 \leq x \leq 2$, $-1 \leq y \leq 4$.)

(b) Find the gradient vector $\nabla f$. Mark on your graph from part (a) the direction of the gradient vector at the points $(1, 1)$ and $(-1, 3)$.

(c) Does the function $f$ have a global maximum or global minimum? Explain your answer.

5. Use a linear approximation to estimate the value of $f(1.05, 2.05)$ where

$$f(x, y) = \sin(x + 2y - 5).$$