Math 107 Final Exam Review

1. Evaluate (10 pts each)

a) $\int_0^1 x^2 \ln(1+x)\, dx$

b) $\int \frac{x^3}{(x^2-1)}\, dx$

c) $\int_0^1 x^3 \cos(\pi x^2)\, dx$

2. Consider the initial value problem y' = 0.5y + 0.3, y(0) = 0.8.

a) (10 pts) Solve the differential equation and find y(1) exactly.
b) (10 pts) Use Taylor polynomials to find the numerical value of y(1) accurate to 5 decimal places. Justify the error.

3. The total number of people infected with a virus often grows like a logistic curve. Suppose that 10 people originally have the virus, and that in the early stages of the virus (with time measured in weeks), the number of people infected is increasing exponentially with k=1.35. It is estimated that, in the long run, approximately 5000 people become infected.

a) (10 pts) Use this information to find a logistic function to model this situation.

b) (10 pts) Assuming that the model is correct, find the time at which 2500 people become infected. .

4. (15 pts) Approximate $\int_0^{.2} \sin(x^3)\, dx$ with a error less than 0.0001 . Justify the error estimate.

5. (15 pts) Evaluate the improper integral

$$\int_0^{\infty} \frac{dx}{x^2+3x+2}$$

6. (15 pts) The lifespan of a light bulb is exponentially distributed with a mean of 100 hours. Find the probability that a light bulb will last at least

75 hours.

7. (15 pts) Let A= (4,3), B=(1, -2)
Using vectors, find the distance from B to the line in the direction of $\vec{OA}$.

8. (10 pts) Find the equation of the tangent line (in parametric form) to $\vec{r}(t) = \ln t\ \vec{i} + t\ \vec{j}$ at $t = 1$.

9. (10 pts) Let $f(x, y) = x^4 + xy^2$. Find the equation of the line normal to the level set $f(x, y) = -2$ at the point (-2,3) .

10. A Boeing 747 flying horizontally at an altitude of 20 km at a speed of 250m/s drops an engine.

a) (10 pts) How long after it drops off does it take for the engine to hit the ground?

b) (10 pts) How far horizontally has the engine traveled when it hits the ground?

11. The depth (in feet) of a pond at the point with coordinates (x, y) is given by $h(x, y) = 2x^2 + 3y^2$ measured in feet.

a) (10 pts) Find the rate of change of $h(x, y)$ in the direction of the vector $\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$ at the point (-1,2).

b) (10 pts) In which direction should a boat positioned at $(-1, 2)$ move for the depth to remain constant?

c) (10 pts) In which direction should a boat positioned at (-1,2) move so that the depth decreases fastest?