FINAL PRACTICE EXAM IV

YI LI

1. Let $f(x, y) = x^2 - y^2$ with constraint function

$$2x + y = 1.$$

Using Lagrange multipliers to find all extrema.

2. Consider the system of linear equations

$$\begin{aligned} 2x - y + 3z &= 3 \\ 2x + y + 4z &= 4 \\ 2x - 3y + 2z &= 2 \end{aligned}$$

Find the augmented matrix of the above system and use it to solve the system.

3. Let $f(x, y) = \sqrt{4 - x^2 - y^2}$.

(a) Find the largest possible domain and the corresponding range of $f(x, y)$.

(b) Compute $f_x(1, 1)$ and $f_y(1, 1)$.

4. Compute

$$\int_0^1 \ln x \, dx.$$

5. Find the global extrema of

$$f(x, y) = x^2 - 3y + y^2, \quad -1 \le x \le 1, \ 0 \le y \le 2.$$

6. Use the partial-fraction method to solve

$$\frac{dy}{dx} = (y - 1)(y - 2)$$

with $y(0) = 0$.

7. Find all candidates for local extrema and use the Hessian matrix to determine the type:

$$f(x, y) = e^{-x^2 - y^2}.$$

8. Suppose that

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$$

(a) Compute $\det A$. Is A invertible?

(b) Suppose that

$$X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Write $AX = B$ as a system of linear equations.

(c) Show that if

$$B = \begin{bmatrix} 3 \\ \frac{9}{2} \end{bmatrix}$$

then $AX = B$ has infinitely many solutions.

9. Solve the given initial-value problem

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

with $x_1(0) = 3$ and $x_2(0) = -1$.

10. Suppose that

$$\frac{dy}{dx} = y(4 - y).$$

(a) Find the equilibria of this differential equation.

(b) Compute the eigenvalues associated with each equilibrium and discuss the stability of the equilibria.

Department of Mathematics, Johns Hopkins University, 3400 N Charles Street, Baltimore, MD 21218, USA

E-mail address: yli@math.jhu.edu