FINAL PRACTICE EXAM III

YI LI

1. Let $f(x,y) = e^{-xy}$ with constraint function
$$x^2 + 4y^2 = 1.$$
Using Lagrange multipliers to find all extrema.

2. Consider the system of linear equations
$$\begin{aligned} 3x + 5y - z &= 10 \\ 2x - y + 3z &= 9 \\ 4x + 2y - 3z &= -1 \end{aligned}$$
Find the augmented matrix of the above system and use it to solve the system.

3. Assume that
$$f(x,y) = \begin{cases} 0, & \text{if } xy \neq 0, \\ 1, & \text{if } xy = 0. \end{cases}$$
(a) Show that $f_x(0,0)$ and $f_y(0,0)$ exist.
(b) $f(x,y)$ is not differentiable at $(0,0)$.

4. Compute
$$\int_0^1 \frac{dx}{(x-1)^{2/3}}.$$

5. Find the absolute maxima and minima of $f(x,y) = x^2 + y^2 + x + 2y$ on the disk $D = \{(x,y) \in \mathbf{R}^2 : x^2 + y^2 \leq 4\}$.

6. Use the partial-fraction method to solve
$$\frac{dy}{dx} = 2y(3-y)$$
with $y(1) = 5$.

7. Find the local extrema of
$$f(x,y) = 2x^2 - xy + y^4, \quad (x,y) \in \mathbf{R}^2.$$

8. Let
$$A = \begin{bmatrix} 2 & 4 \\ -2 & -3 \end{bmatrix}$$
Without explicitly computing the eigenvalues of A, decide whether the real parts of both eigenvalues are negative.

9. Solve the given initial-value problem
$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$
with $x_1(0) = -3$ and $x_2(0) = 1$.

10. Suppose that
$$\frac{dy}{dx} = y(2-y)(y-3).$$
(a) Find the equilibria of this differential equation.

(b) Compute the eigenvalues associated with each equilibrium and discuss the stability of the equilibria.

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