FINAL PRACTICE EXAM II

YI LI

1. Let $f(x,y) = x + y$ with constraint function

$$\frac{1}{x} + \frac{1}{y} = 1, \quad x \neq 0, \ y \neq 0.$$

Using Lagrange multipliers to find all local extrema. Are these global extrema?

2. Consider the system of linear equations

$$\begin{aligned} -2x + 4y - z &= -1 \\ x + 7y + 2z &= -4 \\ 3x - 2y + 3z &= -3 \end{aligned}$$

Find the augmented matrix of the above system and use it to solve the system.

3. Let

$$f(x,y) = \begin{cases} \frac{4xy}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

(a) Does the $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?

(b) Is $f(x,y)$ continuous at $(0,0)$?

4. Determine whether

$$\int_{-\infty}^{\infty} \frac{1}{x^2-1} dx$$

is convergent.

5. Find the absolute maxima and minima of $f(x,y) = x^2 + y^2 + x + 2y$ on the disk $D = \{(x,y) \in \mathbf{R}^2 : x^2 + y^2 \leq 4\}$.

6. Use the partial-fraction method to solve

$$\frac{dy}{dt} = \frac{1}{2}y^2 - 2y$$

with $y(0) = -3$.

7. Find and classify the critical points of

$$f(x,y) = x^3 - 4xy + y, \quad (x,y) \in \mathbf{R}^2.$$

8. Compute the directional derivative of $f(x,y) = ye^{x^2}$ at $(0,2)$ in the direction $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$.

9. Consider the following system of differential equations

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

(a) Show that

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

has the repeated eigenvalues $\lambda_1 = \lambda_2 = 1$.

(b) Show that $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$ are eigenvectors of A and that any vector $\begin{bmatrix}c_1\\c_2\end{bmatrix}$ can be written as

$$\begin{bmatrix}c_1\\c_2\end{bmatrix} = c_1 \begin{bmatrix}1\\0\end{bmatrix} + c_2 \begin{bmatrix}0\\1\end{bmatrix}$$

(c) Show that

$$\mathbf{x}(t) = c_1 e^t \begin{bmatrix}1\\0\end{bmatrix} + c_2 e^t \begin{bmatrix}0\\1\end{bmatrix}$$

is a solution of the above system that satisfies the initial condition $x_1(0) = c_1$ and $x_2(0) = c_2$.

10. Suppose that

$$\frac{dy}{dx} = (4 - y)(5 - y).$$

(a) Find the equilibria of this differential equation.

(b) Compute the eigenvalues associated with each equilibrium and discuss the stability of the equilibria.

Department of Mathematics, Johns Hopkins University, 3400 N Charles Street, Baltimore, MD 21218, USA

E-mail address: yli@math.jhu.edu