FINAL PRACTICE EXAM I

YI LI

1. Determine if the following improper integral converges or diverges. If the integral is convergent compute its value.

$$\int_0^\infty xe^{-x}\,dx.$$

2. Let

$$f(x,y) = \begin{cases} \frac{3x^2y^2}{x^3+y^6}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

(a) Does the $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?
(b) Is $f(x,y)$ continuous at $(0,0)$?
3. Solve the following first order separable initial value problem

$$\frac{dy}{dx} = (y-1)(y-2)$$

with $y(0) = 0$.
4. Consider the system of linear equations

$$\begin{aligned} x_1 - x_2 &= 0 \\ 3x_1 + x_2 - x_3 &= 11 \\ 2x_1 + x_2 + 2x_3 &= 11 \end{aligned}$$

Find the augmented matrix of the above system and use it to solve the system.
5. Consider $f(x,y) = 3xy - x^3 - y^3$.
(a) Locate all critical points of $f(x,y)$.
(b) Classify the critical points of $f(x,y)$ (i.e., determine if they are local maximum/local minimum or saddle point).
(c) Does f have a global maximum or minimum on $\mathbf{R}^2$? Briefly explain!
6. Consider the following system of differential equations

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Solve the following initial value problem with $x_1(0) = 5$ and $x_2(0) = 3$.
7. Find the absolute maxima and minima of $f(x,y) = x^2 + y^2 - 2x + 4$ on the disk $D = \{(x,y) \in \mathbf{R}^2 : x^2 + y^2 \leq 4\}$.
8. Let $f(x,y) = \sqrt{4x^2 + y^2}$ be a function of two variables.
(a) Compute the directional derivative of the function $g(x,y)$ at the point $(-2,4)$ in the direction of $\mathbf{v} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$.
(b) Find the angle between the vectors $\nabla f(-2,4)$ and $\mathbf{v}$.
9. Suppose you wish to enclose a rectangle plot. You have 1600 ft of fencing. Using the material, what are the dimensions of the plot that will have the largest area?

10. Suppose that

$$\frac{dy}{dx} = y(2-y).$$

(a) Find the equilibria of this differential equation.

(b) Compute the eigenvalues associated with each equilibrium and discuss the stability of the equilibria.

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