Exam 1 Solutions

Part I

1. $\int x\ e^{-x}\ dx = x\ (-e^{-x}) + \int e^{-x} \cdot 1\ dx = -e^{-x}(1+x) + C$

2. $\int \frac{1}{x^2-1}\ dx = \int [\frac{\frac{1}{2}}{x-1} - \frac{\frac{1}{2}}{x+1}]\ dx = \frac{1}{2}\ \ln |\frac{x-1}{x+1}| + C$

3. $\int \frac{1}{x^2}\ \ln x\ dx = \ln x(-\frac{1}{x}) + \int \frac{1}{x} \cdot \frac{1}{x}\ dx = -\frac{1}{x}(1+\ln x) + C$

4. We have $N(t) = 100e^{.02t}$ and we want to solve $N(t) = 5000$ or

$$100e^{.02t} = 5000$$

$$e^{.02t} = 50$$

$$.02t = \ln 50$$

$$t = \frac{1}{.02} \ln 50 = 50 \ln 50$$

$$5.\ \ f'(x) = -\frac{1}{2}\ f(x) + 1 = -\frac{1}{2}(f(x) - 2)\ \ ,$$

hence

$$f(x) - 2 = Ce^{-\frac{x}{2}}$$

Using $f(0) = 1$, we find $C = -1$ and $f(x) = 2 - e^{-\frac{x}{2}}$.

Part II

$$6a\quad \cosh x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \ldots + \frac{x^{2n}}{(2n)!} + R_{2n}$$

where

$$|R_{2n}| \leq (\frac{3}{2} + \frac{1}{2})\frac{x^{2n+2}}{(2n+2)!}$$

b. We want to choose n so that

$$2\frac{(.1)^{2n+2}}{(2n+2)!} < 10^{-6}$$

n=1 error $< 2(.0001)/24 = .0000083$
n=2 error $< 2(.000001)/720 = .000000028$
so $\cosh .1 \approx 1 + (.1)^2/2 + (.1)^4/24 = 1.005004$

$$7.\ \ y'(t) = y\ (1 - 2y)\ \ ,\ \ y(0) = 0.1$$

By partial fractions:

$$\frac{1}{y(1-2y)} = \frac{1}{y} + \frac{2}{1-2y}$$

Integration gives:

$$\ln|\frac{y}{1-2y}| = e^t + c$$

$$\frac{y}{1-2y} = Ce^t\ \ , C = \frac{.1}{.8} = \frac{1}{8}$$

$$y(t) = \frac{.125e^t}{.25e^t + 1}$$

b. We see that $L \approx \frac{.125e^t}{.25e^t}$ when t is very large and so

$$L = \frac{.125}{.25} = \frac{1}{2}$$

Finally, when is $y(t) = \frac{L}{2} = \frac{1}{4}$? Exactly when

$$.5e^t = .25e^t + 1$$

$$e^t = 4\ \ ,\ \ t = \ln 4 = 1.3863$$