

Calculus II Fall 2009

Exercises For Exam I

21 May 2009

1.
 - Summarize the methods to solve system of linear equations. (i.e. Reduction of Equations, Substitution, Matrices, Gaussian elimination, Finding Inverses etc.)
 - Give a summary of solutions of linear equations. (When it has unique solution, infinitely many solution etc.)
 - Give a summary of matrix calculations (i.e. Multiplication, addition, finding inverses etc.)
 - Give a summary of stability and equilibrium..

2. Consider the following system of linear equations

$$\begin{aligned}x + 2y + z &= 5 \\x - y - z &= -2 \\y + z &= 2\end{aligned}$$

- (a) Solve the system by reducing the equations.
- (b) Solve the augmented matrix of the system. Reduce the augmented matrix and find the solutions.
- (c) Write the system of equation as $AX = B$ where A , X and B are matrices.
- (d) Find the inverse of A .
- (e) Solve the system by applying A^{-1} .

3. Same question with the following system.

$$\begin{aligned}x + y &= 4 \\2x - y &= -1\end{aligned}$$

4. Same question with the following system.

$$\begin{aligned}x - y &= 4 \\x + 4y &= -6\end{aligned}$$

5. Find the solution set of the following system of equations (if exists).

(a)

$$\begin{aligned}x + 2y - z &= -17 \\x - y - z &= 3\end{aligned}$$

(b)

$$\begin{aligned}x + 2y - z &= -17 \\x - y - z &= 3\end{aligned}$$

(c)

$$\begin{aligned}x + y &= 2 \\4x - 2y - 5z &= 4 \\x - y - 12z &= 3 \\x - y - z &= 3\end{aligned}$$

(d)

$$\begin{aligned}x - y &= 4 \\x + 4y &= -6 \\x - y &= 4 \\x + 4y &= -6\end{aligned}$$

6. Questions 1-6 From Matrix Operations <http://math.jhu.edu/~malan/Fall12009Calculus/MatrixOperations.pdf>

7. Find the eigenvalues and eigenvectors of the following matrices.

$$(a) \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \quad (b) \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \quad (c) \quad C = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

8. Find the equation of the line in the plane passing through $(2, 3)$ and perpendicular to $(1, 1)$.

9. Find the equation of the plane in the space passing through $(2, 3, 4)$ and perpendicular to $(1, 1, -2)$.

10. Solve the following differential equations

$$(a) \quad \frac{dy}{dx} = \sin(x) \text{ where } y(0) = 2.$$

- (b) $\frac{dy}{dx} = y^2x^2$ where $y(1) = 1$.
- (c) $\frac{dy}{dx} = 2y(y - 3)$ where $y(0) = 2$.
- (d) $\frac{dy}{dx} = (y - 1)(y - 2)$ where $y(0) = 1$.

11. Find the equilibrium solutions of the followings. Determine whether the solutions are stable or unstable equilibrium.

- (a) $\frac{dy}{dx} = y$.
- (b) $\frac{dy}{dx} = ye^y$.
- (c) $\frac{dy}{dx} = 2y(y - 3)$.
- (d) $\frac{dy}{dx} = (y - 1)(y - 2)$.

12. Find the limit of following functions.

- (a) $\lim_{(x,y) \rightarrow (1,2)} \frac{x+y}{x^2+y}$.
- (b) $\lim_{(x,y) \rightarrow (2,2)} \frac{x^2e^y-2}{x+y}$.

13. Show that following limits does not exist.

- (a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x+y}{x^2+y}$.
- (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+y^2}{x^2+xy}$.