

Calculus II Fall 2009
Exercises For Final

1.
 - Summarize the methods to solve system of linear equations. (i.e. Reduction of Equations, Substitution, Matrices, Gaussian elimination, Finding Inverses etc.)
 - Give a summary of solutions of linear equations. (When it has unique solution, infinitely many solution etc.)
 - Give a summary of matrix calculations (i.e. Multiplication, addition, finding inverses etc.)
 - Give a summary of stability and equilibrium.
 - Give a summary of partial derivatives, tangent planes, linearization,
 - Summarize how to find max/min.
 - Give a summary of system of linear equations. (How to solve, Stability)
 - Give a summary of Counting. (Permutation, Combination, Multiplication, Combination)
 - Give a summary of Probability, Equally Likely Outcomes
 - Give a summary of Conditional Probability and Independence
 - Give a summary of Discrete Distributions, Mean and variance, Binomial Distribution
 - Give a summary of Continuous Distributions, Mean and variance, Normal Distribution

2. Consider the following system of linear equations

$$\begin{aligned}x + 2y + z &= 5 \\x - y - z &= -2 \\y + z &= 2\end{aligned}$$

- (a) Solve the system by reducing the equations.
- (b) Solve the augmented matrix of the system. Reduce the augmented matrix and find the solutions.
- (c) Write the system of equation as $AX = B$ where A , X and B are matrices.
- (d) Find the inverse of A .

(e) Solve the system by applying A^{-1} .

3. Same question with the following system.

$$\begin{aligned}x + y &= 4 \\2x - y &= -1\end{aligned}$$

4. Same question with the following system.

$$\begin{aligned}x - y &= 4 \\x + 4y &= -6\end{aligned}$$

5. Find the solution set of the following system of equations (if exists).

(a)

$$\begin{aligned}x + 2y - z &= -17 \\x - y - z &= 3\end{aligned}$$

(b)

$$\begin{aligned}x + y &= 2 \\4x - 2y - 5z &= 4 \\x - y - 12z &= 3 \\x - y - z &= 3\end{aligned}$$

(c)

$$\begin{aligned}x - y &= 4 \\x + 4y &= -6\end{aligned}$$

6. Questions 1-6 From Matrix Operations <http://math.jhu.edu/~malan/Fall2009Calculus/MatrixOperations.pdf>

7. Find the eigenvalues and eigenvectors of the following matrices.

$$(a) \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \quad (b) \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \quad (c) \quad C = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

8. Find the equation of the line in the plane passing through $(2, 3)$ and perpendicular to $(1, 1)$.

9. Find the equation of the plane in the space passing through $(2, 3, 4)$ and perpendicular to $(1, 1, -2)$.

10. Solve the following differential equations

(a) $\frac{dy}{dx} = \sin(x)$ where $y(0) = 2$.

(b) $\frac{dy}{dx} = y^2x^2$ where $y(1) = 1$.

(c) $\frac{dy}{dx} = 2y(y - 3)$ where $y(0) = 2$.

(d) $\frac{dy}{dx} = (y - 1)(y - 2)$ where $y(0) = 1$.

11. Find the equilibrium solutions of the followings. Determine whether the solutions are stable or unstable equilibrium.

(a) $\frac{dy}{dx} = y$.

(b) $\frac{dy}{dx} = ye^y$.

(c) $\frac{dy}{dx} = 2y(y - 3)$.

(d) $\frac{dy}{dx} = (y - 1)(y - 2)$.

12. Find the followings

$$\begin{array}{cccccc} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \\ \frac{\partial^3 f}{\partial x^3} & \frac{\partial^3 f}{\partial x^2 \partial y} & \frac{\partial^3 f}{\partial y^3} & \frac{\partial^3 f}{\partial x \partial y^2} & & \end{array}$$

(a) $f(x, y) = x^2 - y^2$

(b) $f(x, y) = x^2e^{x^2-y}$

(c) $f(x, y) = \ln(\sin(x) + \cos(y))$ where $\sin(x) + \cos(y) > 0$

(d) $f(x, y) = x^y, x > 0, y > 0$

13. Find the linearization of f and the equation of the tangent plane the graph of f at $(0, 0)$.

(a) $f(x, y) = x^2 - y^2 + x$

(b) $f(x, y) = x^2 + e^{x^2-y}$

14. Find the Jacobian matrix and the linearization of f at $(0, 0)$

(a) $f(x, y) = \begin{bmatrix} x^2 - y^2 + e^x \\ x^2 - e^{x^2-y} \end{bmatrix}$

$$(b) f(x, y) = \begin{bmatrix} \sin(x + y) \\ \cos(x + y) \end{bmatrix}$$

15. Compute the directional derivative at the given point in the indicated direction.

$$(a) f(x, y) = x^2 - y^2 + e^x \text{ at } (1, 1) \text{ in the direction of } \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$(b) f(x, y) = x^3 - y^3 + x - y \text{ at } (1, 0) \text{ in the direction of } \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

16. In what direction does $f(x, y) = x^3 - y^2 - e^x$ increase most rapidly at $(1, 1)$.

17. In what direction does $f(x, y) = x^2 e^{x^2 - y}$ increase most rapidly at $(1, 0)$.

18. Find the critical points for local max/min and use the Hessian matrix to classify as max/min/saddle.

$$(a) f(x, y) = x^2 - y^2 + x$$

$$(b) f(x, y) = x^2 e^{x^2 - y}$$

19. Solve the system of linear differential equations and discuss the stability.

$$\begin{bmatrix} \frac{\partial x_1}{\partial t} \\ \frac{\partial x_2}{\partial t} \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$(a) A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \quad (b) A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \quad (c) A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$$

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21. page 739 #1-12, 17-22

22. Find all equilibria of each system and discuss stability.

(a)

$$\frac{\partial x_1}{\partial t} = x_1^2 + (x_2 - 1)^2 \quad (1)$$

$$\frac{\partial x_2}{\partial t} = x_2 + x_1 \quad (2)$$

$$\frac{\partial x_1}{\partial t} = x_1 + x_2 \quad (3)$$

$$\frac{\partial x_2}{\partial t} = x_1 - x_2^2 \quad (4)$$

23. A six-sided die with sides labeled 1, 2, 3, 4, 5, and 6 is weighted so that the outcomes 1, 2, 3, 4, and 5 are equally likely to occur. In addition the outcome 6 is three times as likely as the outcome 1. If this die is rolled 5 times, what is the probability of 6 occurs 3 times as outcome? Let X be the random variable that counts the number of 6. Find the probability mass function describing the distribution of X . Find EX , $var(X)$ and the standard deviation.

What is the probability of 4 occurs 4 times as outcome? Let Y be the random variable that counts the number of 6. Find the probability mass function describing the distribution of Y . Find EY , $var(Y)$ and the standard deviation.

24. Consider the coin tossing experiment with a coin weighted so that probability of an outcome of head is twice more likely than tail as outcome. Find the probability that there are fewer than 3 heads tossed if it is tossed 4 times.

25.

Values of X	$Pr[X = x]$
-2	0.1
1	p
3	0.5

- (a) Find p
- (b) Find the expected value and standard deviation.
- (c) Find the cumulative distribution function.
26. A committee of 4 people are going to be chosen from a group of 10. Committee consists of a chairman, a secretary and 2 members. In how many ways we may select this committee.
27. In an experiment, first a dice is rolled and outcome is noted, then a coin is flipped and outcome is noted.
- (a) Write the sample space of outcomes.
- (b) Find the event, E , of getting an odd number and a head.
- (c) Assuming dice and coin are fair, what probability should be assigned to E ?
28. The supreme court of Calculus-Land consists of 8 Judges, 6 of which are female and 2 of which are male. Three of the judges are randomly selected to play on a volleyball team.
- What is the probability that exactly two of the selected judges are female?

29. The WMB motor company produces 20% of its cars at plant A and the remainder at plant B. Of all cars produced at plant A, 30% do not have a spare tire, while 30% of the cars produced at B do not have a spare tire. A WMB car is purchased, and it happens to have a spare tire.

What is the probability that the car was produced at plant A?

30. A sample space S has two independent events, A and B, with $Pr[A] = 0.60$, and $Pr[B|A] = 0.50$. Find $Pr[A^c \cap B^c]$?

31. Assume a box contains 9 red balls, 6 blue balls, and 5 white balls, and that we choose two balls at random from the box.

What is the probability of neither being red given that neither is blue?

32. Assume that there are 5 men and 6 women, and that a committee of 3 is selected at random.

What is the probability that all are male given that all are the same sex?

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34. page 683 # 1-14

35. page 725 #1-58

36. page 739 #1-12, 17-22

37. page 751 #1-10

38. page 770 #7-10

39. page 793 #1-44

40. page 805 #1-40

41. page 818 #1-33

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43. page 885 # 1-20, 25-29, 35